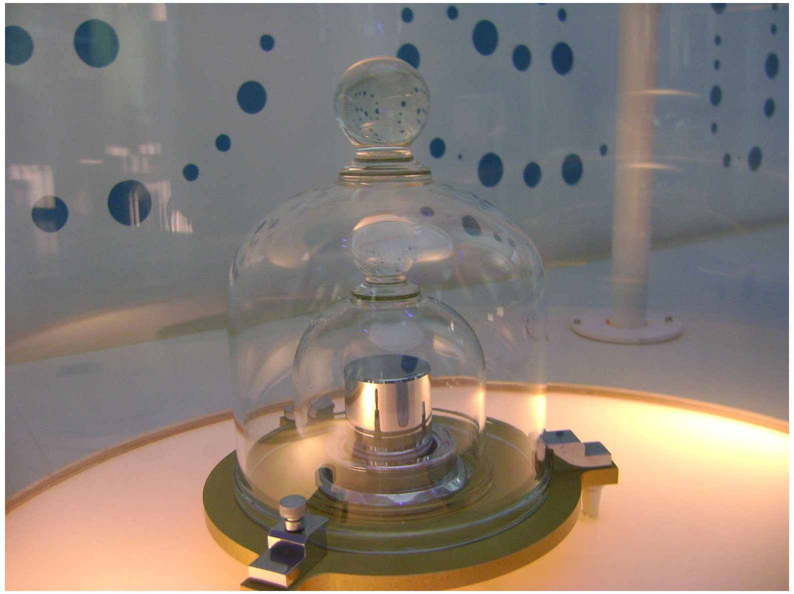
UNITS

INTRODUCTION

Electricity is described in terms of physical quantities (voltage, current, resistance). These all have *units* associated with them. Units may also have a *prefix* associated with it to make dealing with very large (or small) units easier.

The original SI units were designed around mass, length and time. They were not designed with electricity in mind. We will have to learn several new units as well as unit prefixes to make recording electrical measurements practical.



BASE UNITS

SI UNITS

There are *seven* SI base units. Every single other unit can be expressed in terms of these base units.

As of 2019, the units are based on fixing the values of various physical constants.

The diagram on the right shows these base units and their relationships.

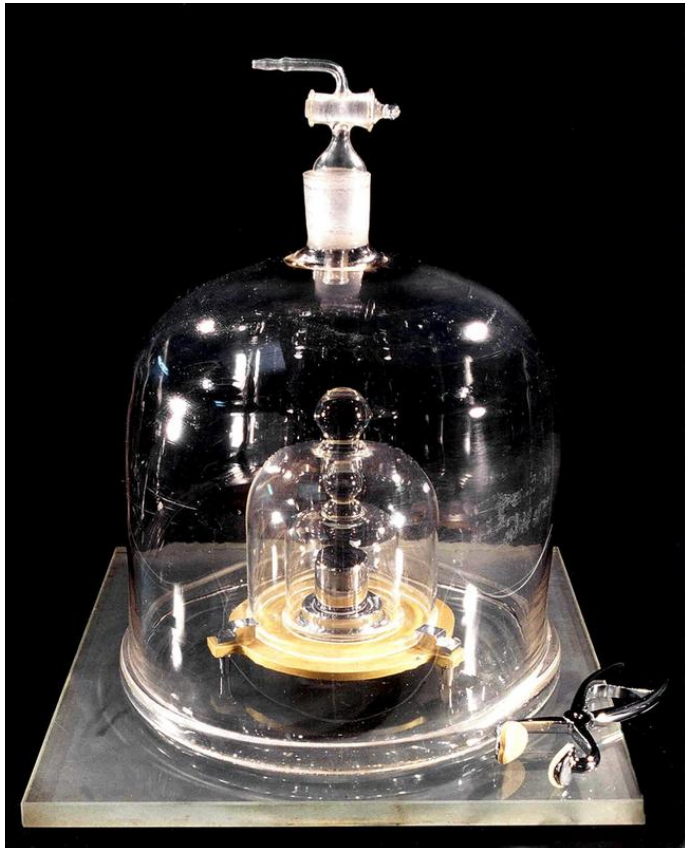
MASS - KILOGRAM

The basic unit of mass is the *kilogram*, or **kg**. This is the only base unit that has a prefix built into it (more on prefixes later).

A kilogram is approximately the same mass as a litre of water.

This mass was copied into a platinum-iridium cylinder about 38 mm long and wide (Big K).

The kilogram was based on a physical object until 2019. *Only one Big K ever existed!!!*



MASS – KILOGRAM EXAMPLES

A human has a mass of ~70 kg.

An electron has a mass of 9.109×10^{-31} kg.

The earth has a mass of 5.972×10^{24} kg.

The sun has a mass of 1.989×10^{30} kg.

LENGTH – METRE (OR METER IN USA)

The unit of length is the *metre*, or **m**.

A metre is was originally defined as approximately $\frac{1}{40000000}$ of the distance around the earth.

It then became the distance between two fine lines on a platinum-iridium ruler.

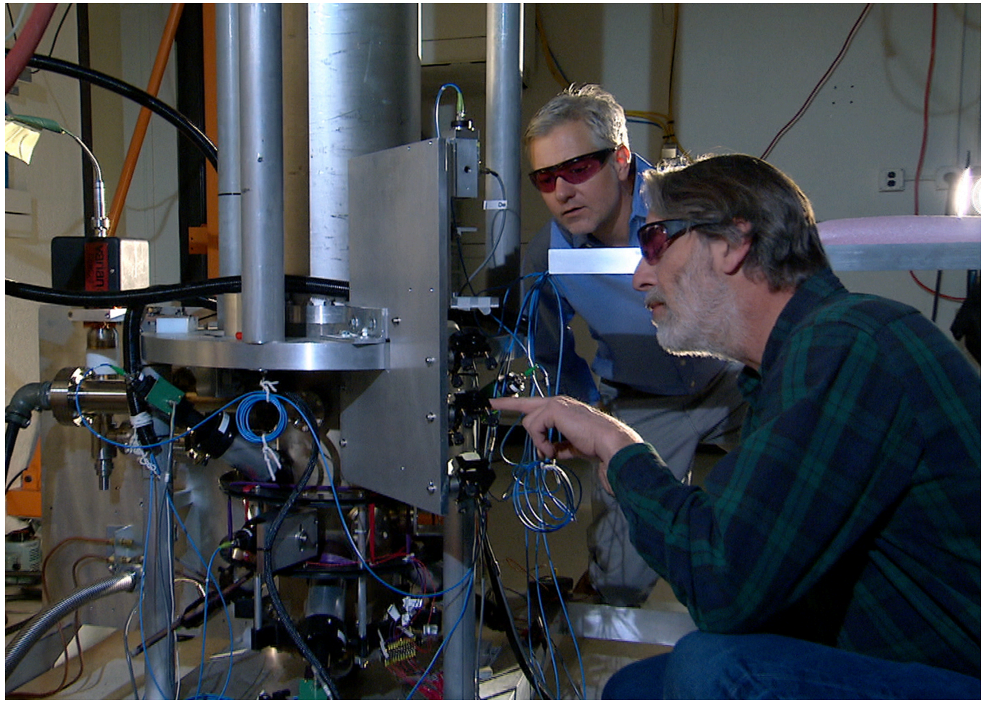
It's now defined as the distance travelled by light in a vacuum in exactly $\frac{1}{299792458}$ of a second.

TIME - SECOND

The base unit of time is the *second*, or **s**.

The second is approximately equal to $\frac{1}{86400}$ of a mean solar day. This is the time it takes for the sun to return to its same apparent position in the sky from one day to the next. This is also 24 hours at 3600 seconds per hour.

The current second is defined as "9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom". You will be required to recite this definition in your first test (joke).



ELECTRIC CURRENT – AMPERE (AMP)

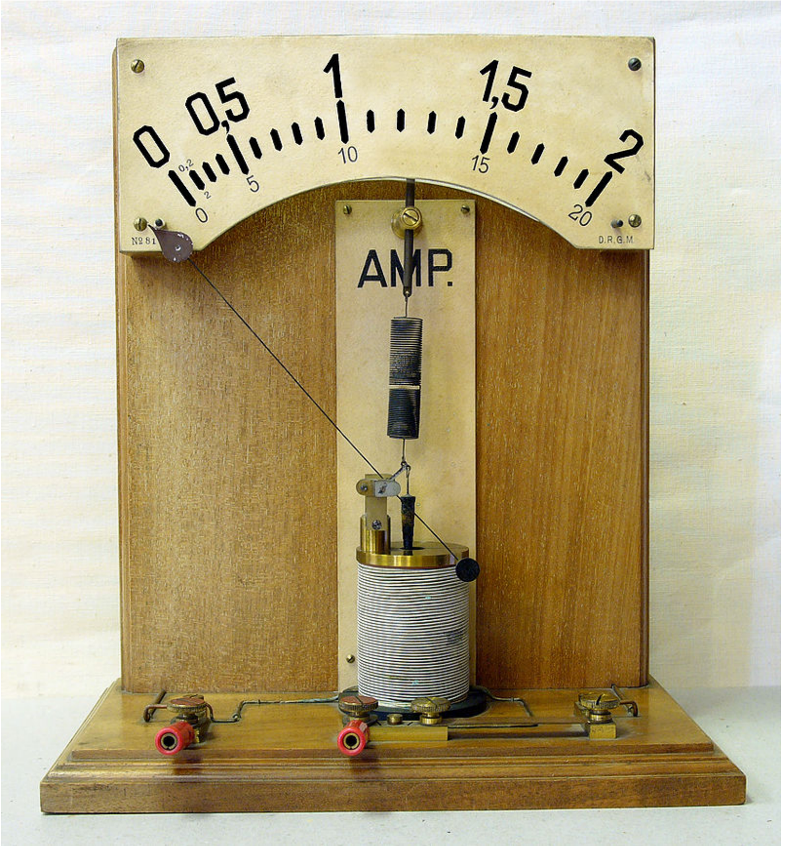
The base unit of time is the *ampere*, or **A**.

This unit describes the number of charged particles passing a point every second.

The definition is based on the definition of the *elementary charge*, or the electric charge that a single electron (or proton) holds.

An amp is defined as the flow of $\frac{1}{1.602176634\times10^{-19}}$ elementary charges, or

6,241,509,074,460,76(3,000) electrons past a given point per second. The numbers in brackets are digits that I couldn't fully calculate.



TEMPERATURE - KELVIN

The base unit of time is the *kelvin*, or **K.**

The kelvin has its base at *absolute zero*, when it's so cold all atoms stop moving.

The *degree Celsius* is a *derived unit* from the kelvin. It has the symbol **°C.**

1 degree Celsius and 1 kelvin are about $\frac{1}{100}$ the temperature difference between the melting and boiling points of water at sea level.

TEMPERATURE - EXAMPLES

Absolute zero is -273.15°C or 0 K.

The melting point of ice is 0°C or 273.15 K.

The boiling point of water is 100°C or 373.15 K.

Room temperature is about 20°C or 293.15 K.

The "surface" of the sun (the part that we see as a globe) has a temperature of 5700 K.

LIGHT OUTPUT - CANDELA

The base unit of light output is the *candela*, or **cd**.

The candela relates the energy and power at different wavelengths in the light, to what our eyes perceive as light.

It's approximately the angular light intensity of a single candle.

A light source emitting 1 cd will produce 1 lumen of total light flux on 1 m^2 'cap section' of a sphere of 1 m radius (1 steradian of solid angle).

The candela, and its derived units lumens and lux, are used in lighting calculations.

LIGHT OUTPUT - CANDELA

A light source emitting 1 cd equally in all directions emits exactly 4π (~12.57) *lumens* (**lm**) over a sphere.

i.e. 1 lm = 4π cd in all directions over a sphere.

A 60 watt incandescent bulb emits about 600 lumens.

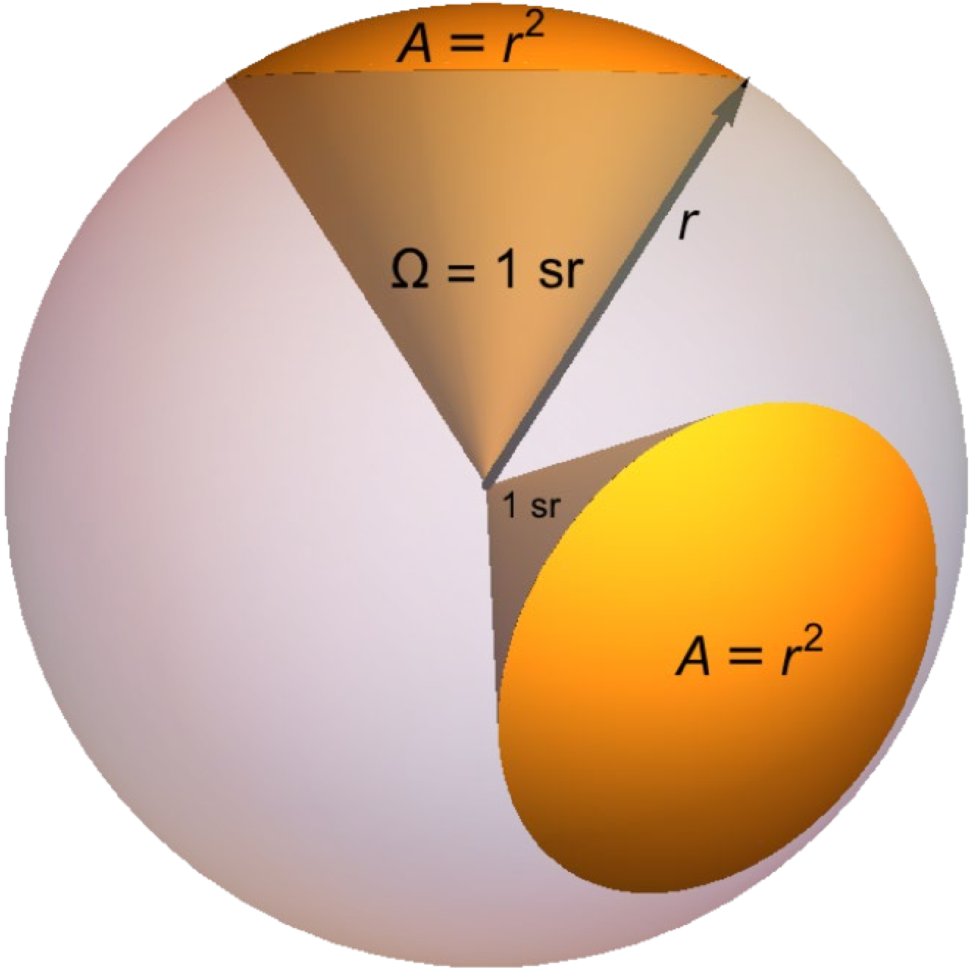
The unit *lux* (**lx**) is based on the lumens per area, and is used to describe the intensity of illumination.

1 lx = 1 lm m^{-2}

Bright sunlight is about 100000 lx.

A typical living room at night is about 50 lx.

The full moon is about 0.3 lx.



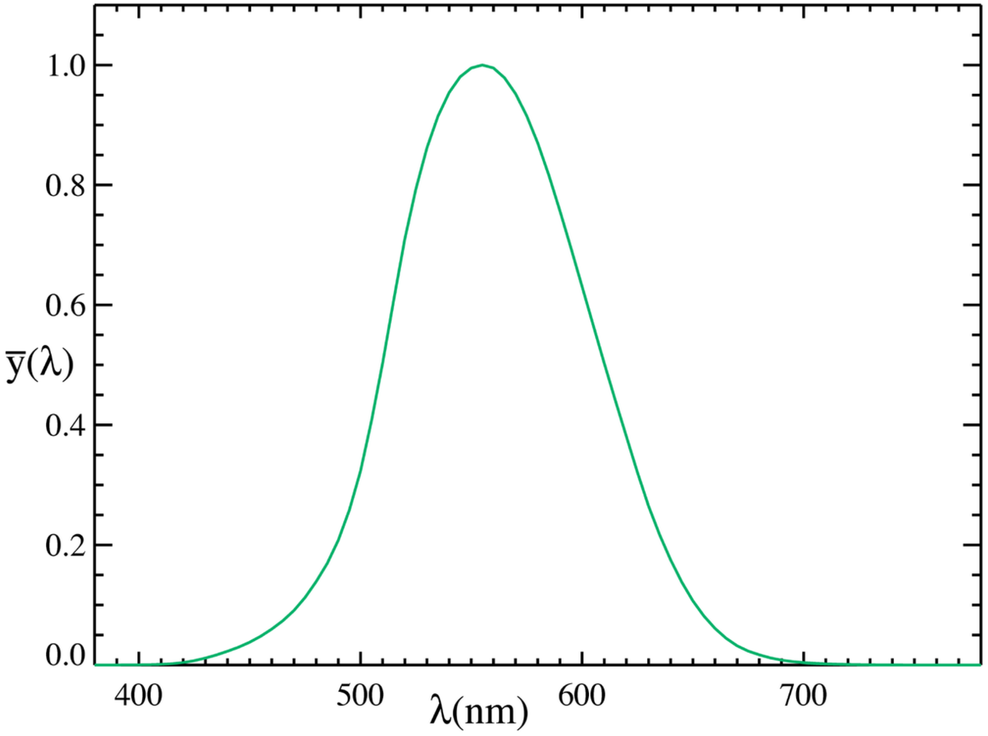
LIGHT OUTPUT – LUMENS AND WATTS

The relationship between input power and output lumens depends on the wavelength.

The graph on the right shows the perceived brightness compared to light wavelength. Violet is on the left, red is on the right. The peak is at 555 nm wavelength, and corresponds to green.

At 555 nm, 1 watt of emitted light power will produce 683 lumens of illumination.

Other wavelengths will require higher power outputs to provide the same level of illumination.



LIGHT OUTPUT - EXAMPLES

Different light sources emit different amounts of light per unit power.

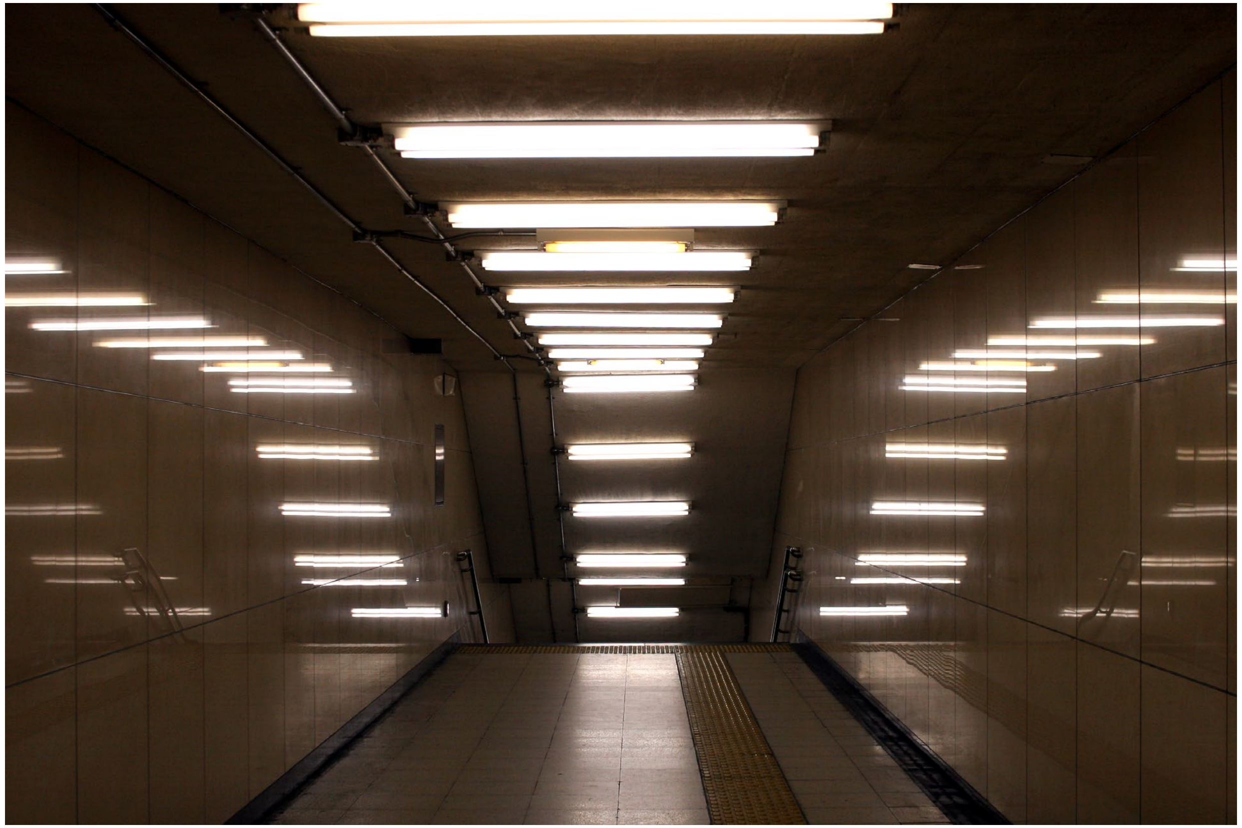
Incandescent light bulbs are about 14 lumens per watt (lm W^{-1}).

Fluorescent tubes are about 63 lm W^{-1}.

LED bulbs are about 74 lm W^{-1}.

The sun is about 93 lm W^{-1}.

Sodium lamps are about 150 lm W^{-1}, but are not white light sources.



DERIVED UNITS

DERIVED UNITS

All SI units are derived from these base units.

There are other units that may have different names, but they can always be related to these base units.

Derived units are used because it can be unwieldy to use base units only, particularly with electrical units.

E.g.

An AA battery has an electromotive force of 1 $kg\ m^2\ s^{-3}\ A^{-1}$. Compare V as a unit.

A resistor has a value of 1 $kg\ m^2\ s^{-3}\ A^{-2}$. Compare Ω as a unit.

The electric permittivity of free space (ε_0) is $8.85 \times 10^{-12}\ kg^{-1}\ m^{-3}\ s^4\ A^2$. Compare $F\ m^{-1}$ as a unit.

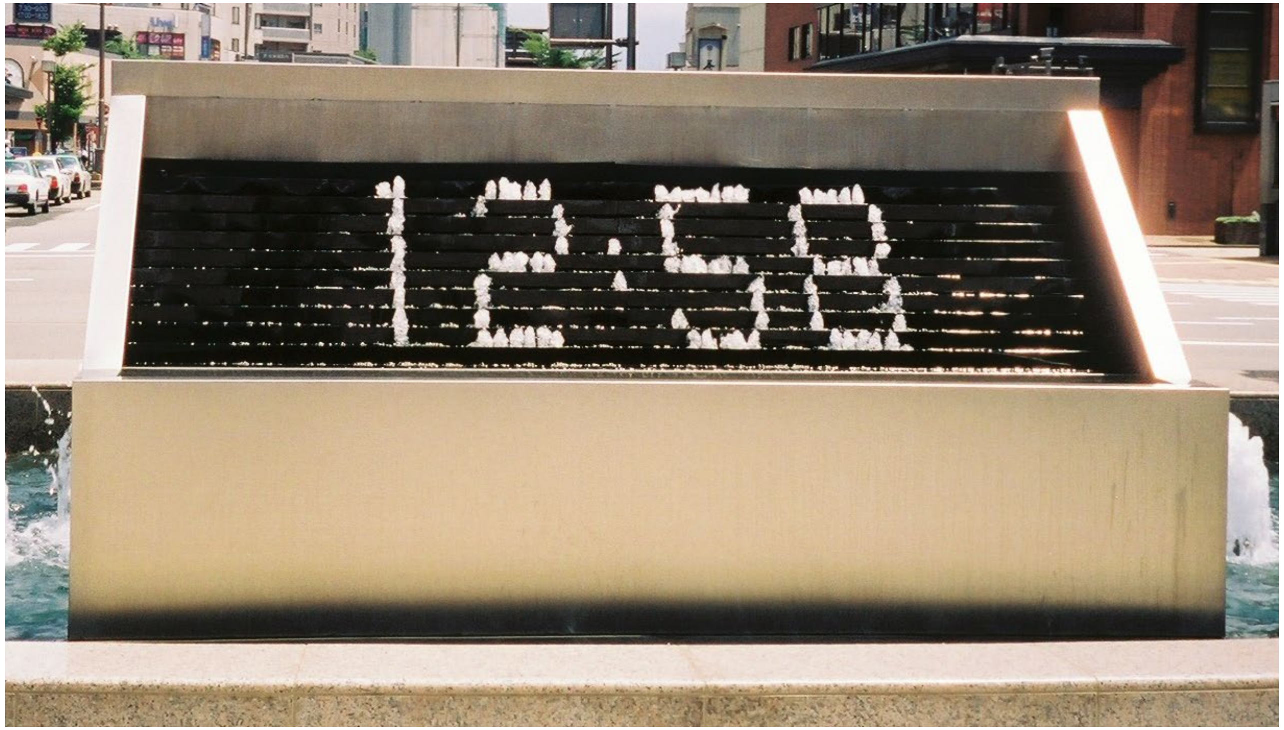
The magnetic permeability of free space (μ_0) is about $4\pi \times 10^{-7}\ kg\ m\ s^{-2}\ A^{-2}$. Compare $H\ m^{-1}$ as a unit.

DERIVED UNITS - TIME

These are two main derived units from time:

Minute (**min**) 1 min = 60 s

Hour (**h or hr**) 1 h = 60 min = 3600 s



FREQUENCY

Frequency is given in *hertz*, or **Hz**.

1 Hz = 1 s^{-1}

The alternating current mains supply in New Zealand has a frequency of 50 Hz.

Human hearing range is from 20 Hz to 20000 Hz.

FM radio is in the frequency range of 88000000 – 108000000 Hz (88 – 108 MHz).

Light has a frequency of about 430000000000000 Hz (red) to 750000000000000 Hz (violet) (430 – 750 THz).

DERIVED UNIT – SPEED AND ACCELERATION

Speed is the length travelled per unit time.

The unit is expressed as m/s or m s^{-1}. An object travelling 1 m s^{-1} will travel 1 metre in 1 second.

The speed of light in a vacuum is exactly 299792458 m s^{-1}.

Derived units include km/h or km h^{-1}.

1 km h^{-1} = 0.27778 m s^{-1}.

Acceleration is the change of speed over time, or m s^{-2}. The earth's gravity produces an acceleration of 9.81 m s^{-2}.

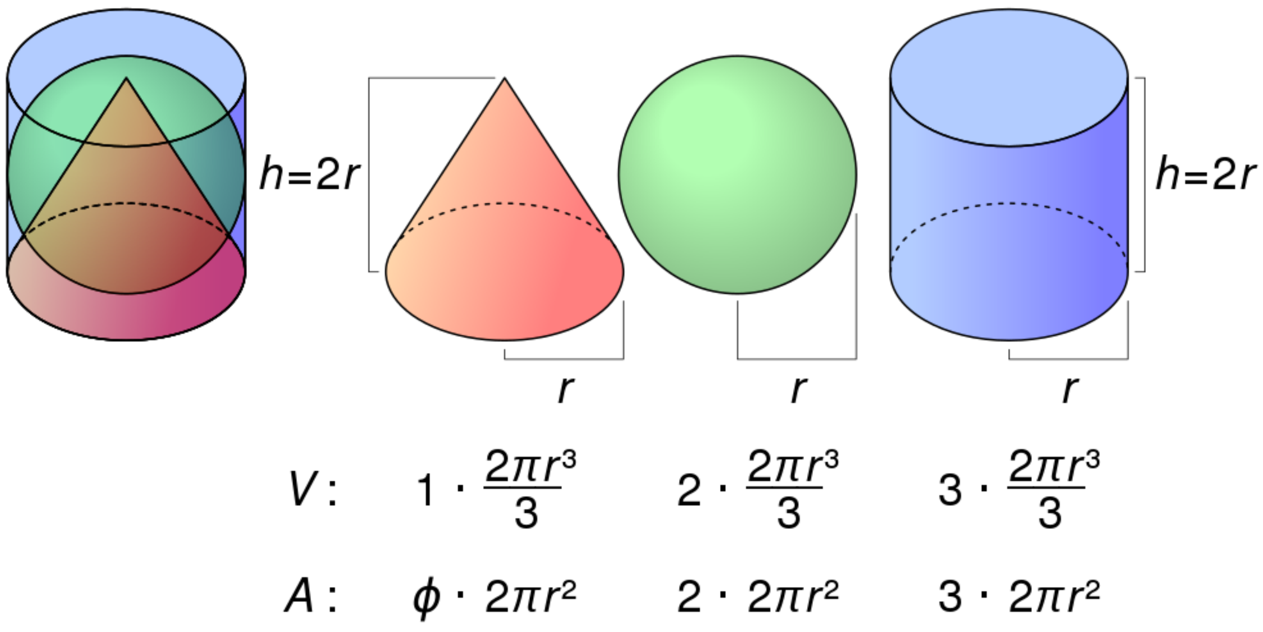
DERIVED UNITS – AREA, VOLUME

Area is given in *square metres*, or **m^2**.

Volume is given in *cubic metres*, or **m^3**.

A derived unit from area is the *hectare*, or ***ha***. 1 ha is the area of a square 100 metres on the side, or 10000 m^2.

A derived unit from volume is the *litre*, symbol **l** or **L**. 1 L is the volume of a cube 0.1 m on the side, or 0.001 m^3.



ENERGY - JOULE

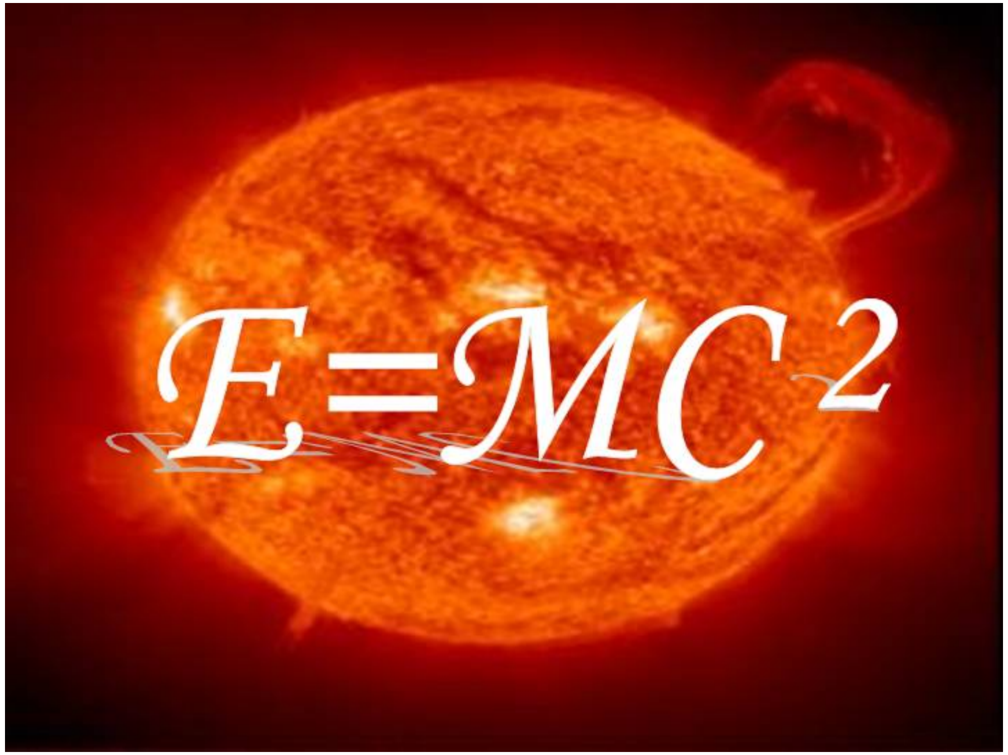
The SI unit of energy is the *Joule*, or **J**.

The unit is based on mass multiplied by speed of light squared.

$1\ J = 1\ kg\ m^2\ s^{-2}$.

Note that there is no amps in energy! Electricity still deals with energy, but the electrical parts cancel themselves out.

As an example, a heater consuming 1000 watts releases 1000 joules of heat energy every second.



POWER - WATT

The SI unit of energy is the *Watt*, or **W**.

The unit is based on energy per unit time.

1 W = 1 J s^{-1} = 1 kg m^2 s^{-3}.

Note that there is no amps in power! Electricity still deals with power, but the electrical parts cancel themselves out.

ELECTRICAL UNITS

ENERGY – WATT-HOUR

1 Joule is quite a small amount of electrical energy.

Electrical energy is often measured in watt-hours, or kilowatt-hours.

1 Wh = 3600 J

1 kWh = 3600000 J

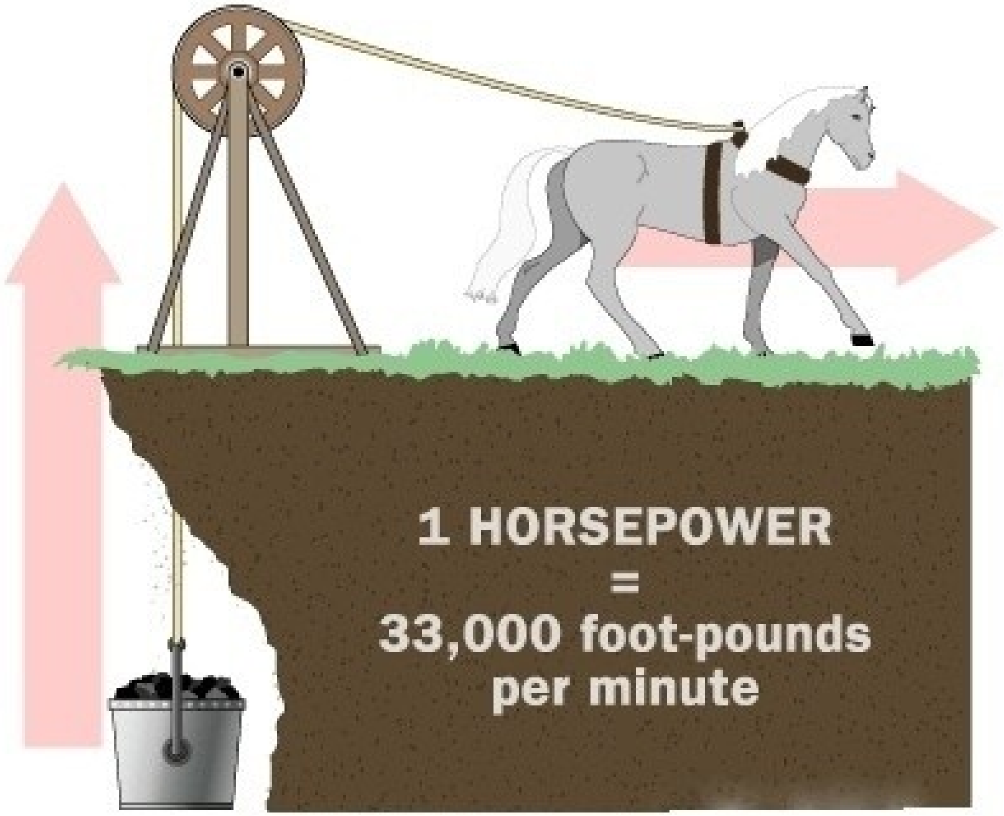
Electricity to the home consumer is sold in units of kilowatt-hours.

POWER – HORSEPOWER

James Watt devised the *horsepower* (**hp**) to compare the output of his steam engines to a horse.

1 hp = 746 W

Many electric motors (especially older motors) are rated in horsepower of mechanical work output.



ELECTROMOTIVE FORCE - VOLT

The SI unit of voltage is the *Volt*, or **V**.

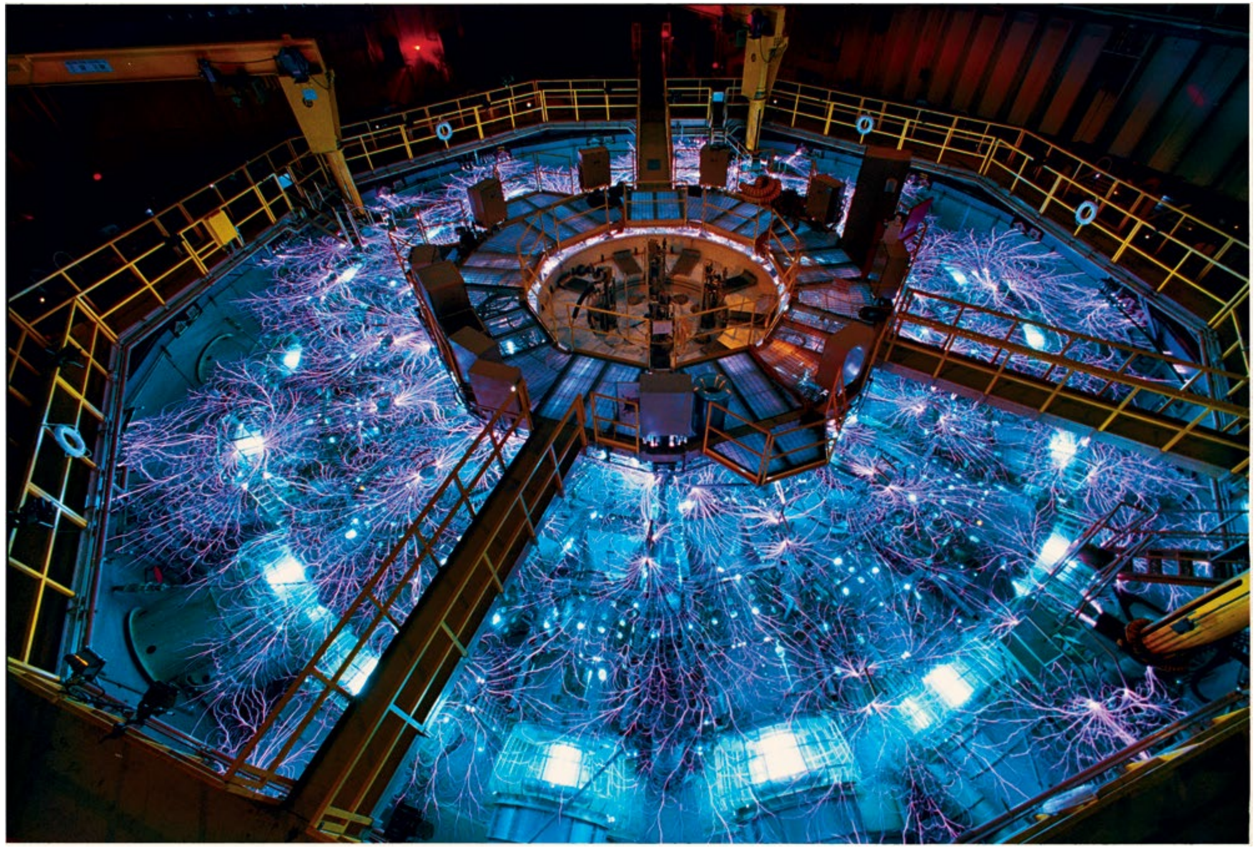
The unit is based on the amount of energy released per unit charge.

$1\ V = 1\ J\ C^{-1}$.

A charge transfer of 1 coulomb passed through a potential difference of 1 volt will transfer 1 joule of energy.

$1\ V = 1\ W\ A^{-1}$.

A car battery has a terminal voltage of 12 V. The mains voltage is 230 V.



RESISTANCE - OHM

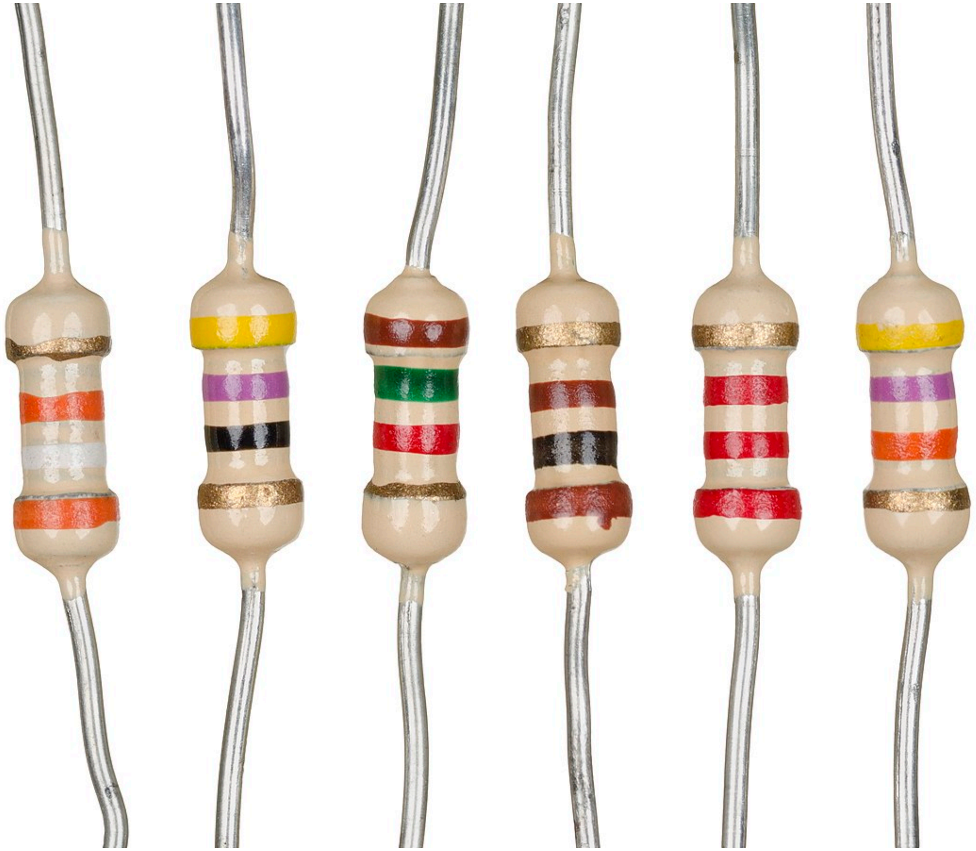
The SI unit of resistance is the *Ohm*, or **Ω.**

The unit is based on the electromotive force required per unit of current.

$1\ \Omega = 1\ V\ A^{-1}$.

A current of 1 amp flowing through a resistor of 1 ohm will produce a voltage drop of 1 V.

Resistance can vary from less than 0.1 Ω (good conductor) to over 1000000000 Ω (good insulator).



ELECTRIC CHARGE - COULOMB

The SI unit of electric charge is the *Coulomb*, or **C**.

The unit is based on electric current multiplied by time.

1 C = 1 A s.

A current of 1 amp means that 1 coulomb of charge is passing past that point every second.

Each electron and proton has a charge of $1.60217662 \times 10^{-19}$ C.

CAPACITANCE - FARAD

The SI unit of capacitance is the *Farad*, or **F**.

A capacitor of 1 farad will store 1 coulomb of charge for every volt across it.

$1\ F = 1\ C\ V^{-1}$

A farad is quite a large unit. Most capacitors used in wiring have values in the microfarad range.

MAGNETIC FLUX - WEBER

The SI unit of total magnetic flux is the *Weber*, or **Wb.**

1 Wb = 1 V s

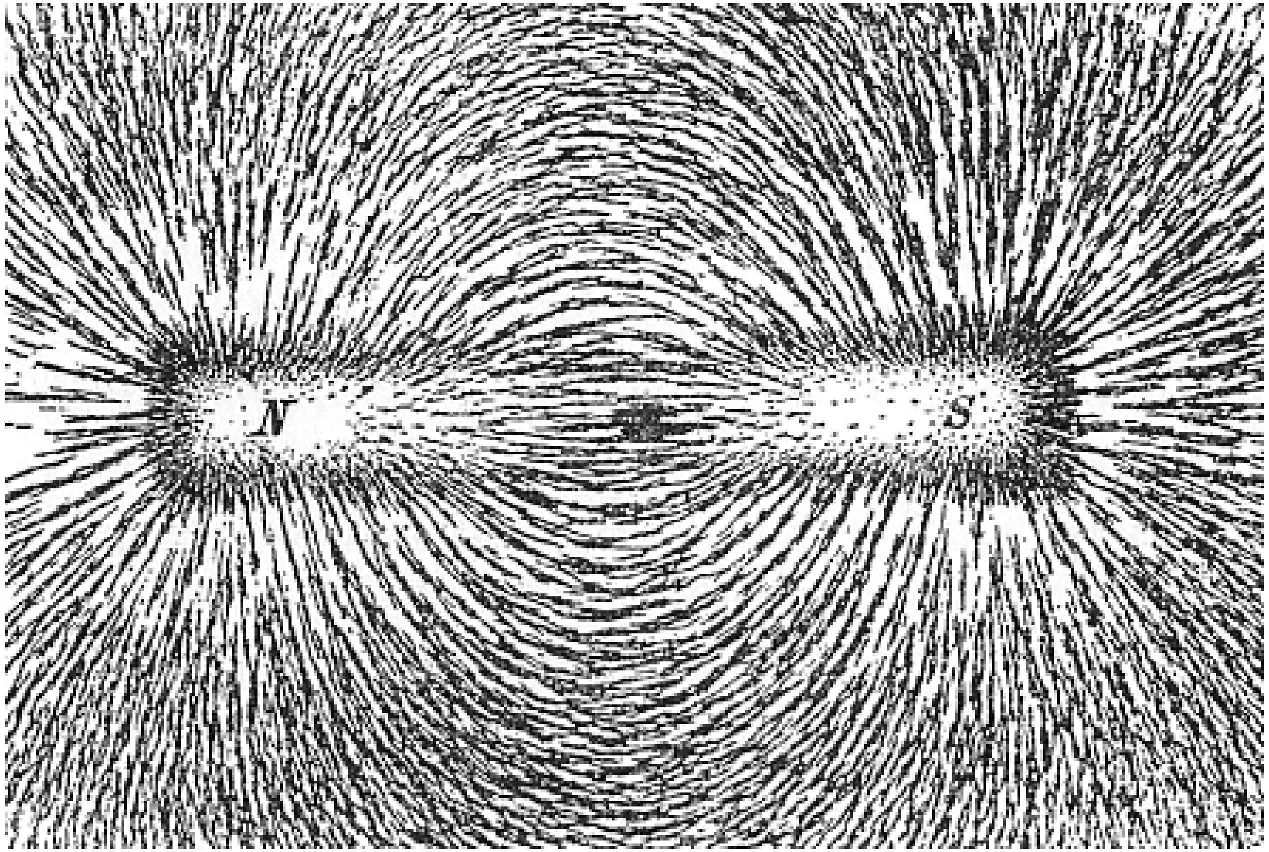
The total magnetic flux is used to derive the *magnetic flux density* and *inductance.*

MAGNETIC FLUX DENSITY - TESLA

The SI unit of total magnetic flux is the *Tesla*, or **T**. It is the magnetic flux density per unit area.

$1 \text{ T} = 1 \text{ Wb m}^{-2}$

Steel used in transformers can withstand a magnetic flux density of about 1.5 T.



INDUCTANCE - HENRY

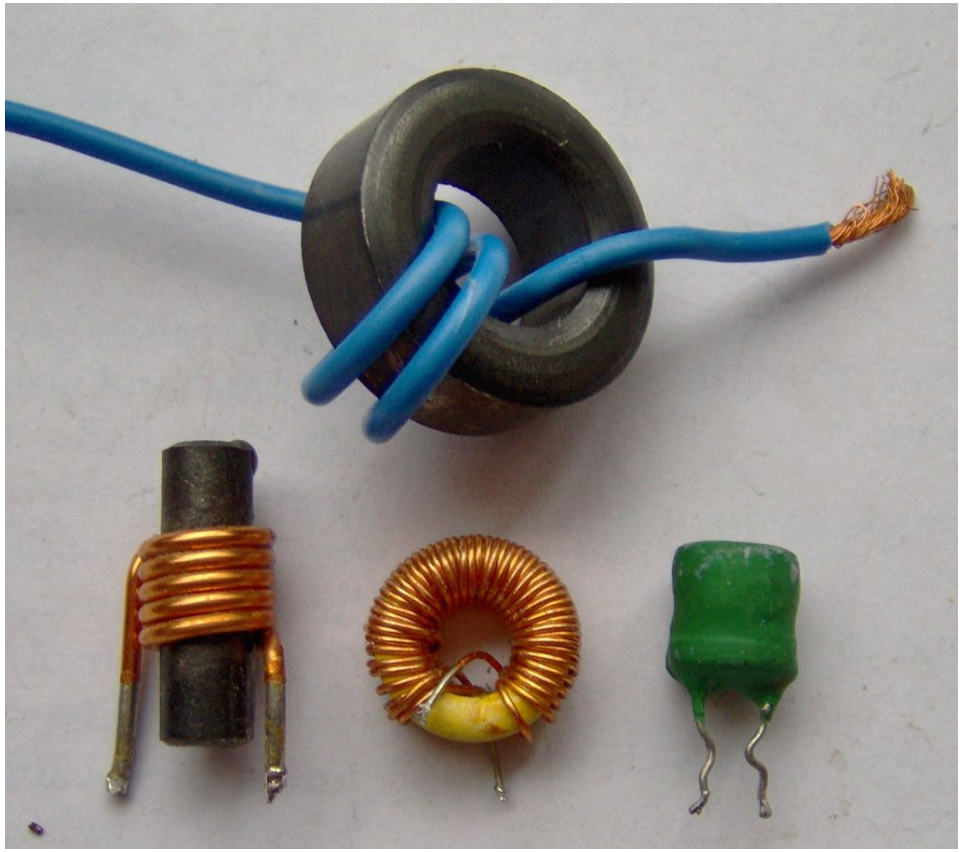
The SI unit of inductance is the *Henry*, or **H**.

An inductor with 1 henry will product 1 Wb of magnetic flux for every amp of current flow.

$1\ H = 1\ Wb\ A^{-1}$.

Another way of thinking of inductance is that if you put 1 volt across a 1 henry inductor, the current will change by 1 amp per second.

$1\ H = 1\ V\ (A\ s^{-1})^{-1} = 1\ V\ s\ A^{-1}$



UNIT PREFIXES

PREFIXES

Units have prefixes to simplify entering very large or small units.

E.g. A motor starting capacitor may have a capacitance of 0.00022 F.

This is quite unwieldy, so prefixes are used to simplify the expression. So 0.00022 F → 22 μF.

The minimum insulation resistance in an electrical installation is 1 MΩ.

Prefixes:

G – giga – 10^9 – 1000000000

M – mega – 10^6 – 1000000

k – kilo – 10^3 – 1000

c – centi – 10^{-2} – 0.01

m – milli – 10^{-3} – 0.001

μ – micro – 10^{-6} – 0.000001

n – nano – 10^{-9} – 0.000000001

p – pico – 10^{-12} – 0.000000000001

CONVERTING TO AND FROM PREFIXED VALUES

Your calculator may have 'Engineering Notation' that works with exponents that are multiples of 3. Otherwise, the prefix can be found by inspection.

E.g. Your calculator displays 0.00001956 as an answer for a capacitance in farads.

In scientific notation, this is 1.956×10^{-5} F.

In engineering notation, this is 19.56×10^{-6} F.

Engineering notation is easy, it is clear from the 10^{-6} that the answer is 19.56 μF.

You'll need to understand this, as many meters may display their results using prefixed units.

This is particularly true of ohm meters, as they have to measure over many orders of magnitude.

CONVERTING TO AND FROM PREFIXED VALUES – 'NATURAL NOTATION'

For 'natural' notation, numbers less than one:

Move the decimal point to the right in steps of 3 (or multiply by 1000) until there is at least one number before the decimal point. Each time, move one 'down' the prefix list.

0.00001956 F → 0000.01956 mF → 0000019.56 μF → **19.56 μF**

0.00000012 F → 0.00012 mF → 0.12 μF → **120 nF** (note you have to add trailing zeros due to multiplying by 1000)

For 'natural' notation, numbers more than one:

Move the decimal point to the left in steps of 3 (or divide by 1000) until there are fewer than three numbers before the decimal point. Each time, move one 'up' the prefix list.

22000000000 Ω → 22000000.000 kΩ → 22000.000000 MΩ → 22.0000000 GΩ → **22 GΩ**

350000 V → **350 kV**

CONVERTING TO AND FROM PREFIXED VALUES – SCIENTIFIC NOTATION

For scientific notation, numbers less than one:

The exponent will be a negative number. Keep subtracting 1 from the exponent until it is a multiple of 3, while shifting the decimal place one to the right on the significand (the digits of the number shown on the calculator). Find the prefix that corresponds to that 'multiple-of-3' exponent.

1.956×10^{-5} F → **19.56×10^{-6} F** → 19.56 μF

1.2×10^{-7} F → 12×10^{-8} F → **120×10^{-9} F** → 120 nF (note you have to add trailing zeros due to multiplying by 10)

For scientific notation, numbers more than one:

The procedure is the same.

2.2×10^{10} Ω → **22×10^{9} Ω** → 22 GΩ

3.5×10^{5} V → 35×10^{4} V → **350×10^{3} V** → 350 kV

WORKING WITH UNITS

ADDING AND SUBTRACTING UNITS

Units can *only* be added or subtracted *if they are the same type of unit.*

Watch for prefixes! It is generally easier to convert all prefixed units to quantities without prefixes.

Do not mix units when adding or subtracting!

Your calculator will not help you with this.

1 Ω + 1 Ω = 2 Ω

1 kΩ + 1 Ω = 1.001 kΩ = 1001 Ω

1 Ω + 1 mΩ = 1.001 Ω = 1001 mΩ

1 Ω + 1 V = (no physical meaning)

MULTIPLYING AND DIVIDING UNITS

The unit of a multiplied, divided or quantity raised to a power follows the same rules as exponents.

You might be able to substitute the calculated unit for a simpler or more physically meaningful one.

Dividing by a prefixed unit "inverts" the prefix.

A prefix is "included" in a unit raised to a power.

$1\ \mathrm{V} \div 1\ \mathrm{A} = 1\ \mathrm{V\ A^{-1}} = 1\ \Omega$

$1\ \mathrm{V} \div 1\ \mathrm{mA} = 1\ \mathrm{kV\ A^{-1}} = 1\ \mathrm{k\Omega}$

$1\ \mathrm{mm^2} \neq 0.001\ \mathrm{m^2}$

$1\ \mathrm{mm^2} = 0.000001\ \mathrm{m^2}$

$1\ \mathrm{mm^3} \neq 0.\ 001\ \mathrm{m^3}$

$1\ \mathrm{mm^3} = 0.000000001\ \mathrm{m^3}$

$1\ \mathrm{A} \div 1\ \mathrm{mm^2} = 1\ \mathrm{A} \div 0.000001\ \mathrm{m^2} = 1000000\ \mathrm{A} \div 1\ \mathrm{m^2} = 1\ \mathrm{MA\ m^{-2}}$

WORKING WITH UNITS - EXAMPLE

Find the resistance of a 100 m length of 2.5 mm^2 cross-sectional area annealed copper wire.

ρ = 17.2 nΩm

A = 2.5 mm^2

l = 100 m

The equation for the total resistance in this scenario is:

$$R = \frac{\rho l}{A}$$

WORKING WITH UNITS – EXAMPLE 1

Step 1: Eliminate the prefixes on all units.

$\rho = 1.72 \times 10^{-8}$ Ωm

$A = 2.5 \times 10^{-6}$ m^2

$l = 100$ m

Step 2: Plug into equation

$$R = \frac{1.72\times10^{-8}\times100}{2.5\times10^{-6}}$$

Step 3: Remove as many powers of 10 as you can and cancel.

$$R = \frac{1.72\times10^{-6}\times10^{6}}{2.5}$$

Step 4: Get final answer

$$R = \frac{1.72}{2.5} = 0.688\ \Omega$$

Step 5: Check units "line up" as expected.

$$\Omega = \frac{\Omega\mathrm{m}\times\mathrm{m}}{\mathrm{m}^2}$$