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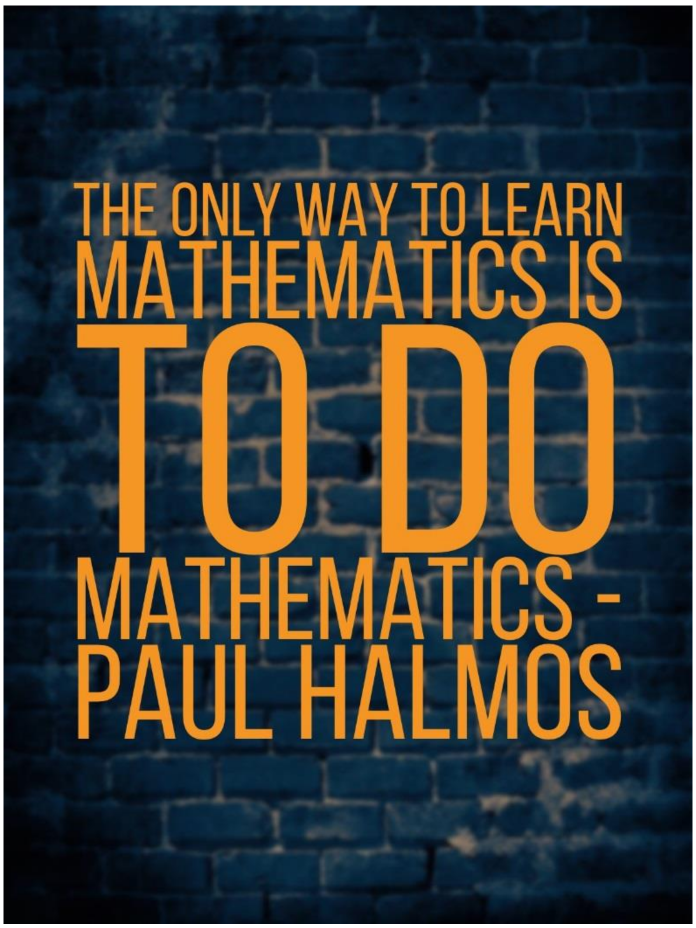
Student Mathematics workbook 2020



Student name

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Introduction

This publication is intended as reference and revision material for the mathematics element of unit 15847. This is a mandatory unit to pass at level 2 of the National Certificate in Electrical Engineering. The seven areas of mathematics covered in this publication provide the skill set needed to study at level 3.

The topics covered are

Order of operations
Ratios
Engineering SI units
Square and square root exponents
Areas and volumes
Trigonometry
Equation transposition

This material is mostly covered in the year 10 college mathematics curriculum. Success at NCEA L1 mathematics should require only revision and practice exercises to attain the 80% pass mark required.

Some students will have had many years away from mathematics study which will require more effort to master the required skills.
Some students may not have understood these skills while at college.
You need to understand where your current level of competence is to start with and the skill gap you need to bridge.
Each student will have a different journey this year.
There will be opportunities to measure where your individual level is at the start of the year and at intervals during.

The approach to delivery of this unit is to teach each skill set in class as a tutorial, answering any queries as they arise.
Practice exercises will be undertaken during class time and some set for homework.
The mathematics unit will be spread over the year to enable enough time for all students to master these necessary skills.

There is extra tuition available from Weltec for students requiring extra study.
Leaving this option, should you need it, until the final stages of the year is not a good idea.

Maths is a subject where you “lose it when you don’t use it”, so constant revision all year is better than a crash effort at the end. If you are not sure of any point, ask!
A small gap in a mathematics sequence can grow into a total misunderstanding if left.

Values, accuracy and units

A **value** tells us **how much.**
This quantifies what we have.
Some numbers are estimates, others are exact.
Sometimes numbers after a calculation never reach an end conclusion, so in our engineering world we need to define the level of accuracy we will work to.

Unless otherwise stated we will define and calculate values to **3 significant figures**.
A zero is significant where it is found between non zero digits.

In practice this means that

23,567	will round up to	**23,6**00
7,824	will round down to	**7,82**0
0.0004567	will round up to	0.000**457**
20,004,000	will round down to	**20,0**00,000
45	will be defined as	**45.0**
0.045	will be defined as	0.0**450**

A **unit** tells us **what** we are valuing.
It is meaningless to know how much, if we don't know what of.

For example,,if you buy a litre of petrol for 50, is it a good deal?
At 50 cents it is.
At 50 dollars it isn't.

Therefore your working and answers will be valued to 3 significant figures and defined as a unit for all your maths exercises

Showing working and estimating

Some work will require entering data into your calculator and copying down your answer. If this is the case a coarse **estimate** should be written down to check if you have used your calculator correctly.

Most of your work will require **line by line working**, where only one operation is performed per line. This allows you to review and check your calculations and allows your tutor to quickly see where you may have made an error.

Electricians are required by law to **check and test** their work, practice that concept with your maths

Order of operations

There are 6 basic mathematical tasks.

Brackets - operations within brackets must be completed first

Exponents - second task

Division
Multiplication } thirdly left to right, as either of these operations appears in sequence

Addition
Subtraction } and finally left to right, as either of these operations appears in sequence

The term **BEDMAS** helps you remember the order of operations
This is the order that these tasks and operations **must** be performed in – or your answer will likely be wrong!

Your Casio fx-82MS calculator will perform this order for you automatically, if you enter the information correctly, so why bother knowing the order?

When you transform equations, which you will need to do later in your electrical theory, this order (in reverse) is used to guide you through rearranging the equations you use. Your calculator cannot do this for you. When you check your work and estimate an answer you should be able to do this manually.

The example below shows how the order of operations works

$\mathbf{10 + (4^2 + 2) \div 9 - 4}$

The exponent inside the brackets is calculated first

$\mathbf{10 + (4^2 + 2) \div 9 - 4 = 10 + (16 + 2) \div 9 - 4}$

The addition inside the brackets is next and removes the need for the brackets

$\mathbf{10 + (16 + 2) \div 9 - 4 = 10 + 18 \div 9 - 4}$

The division is the next operation

$\mathbf{10 + 18 \div 9 - 4 = 10 + 2 - 4}$

Finally the addition and subtraction left to right

$\mathbf{10 + 2 - 4 = 8}$

Bedmas homework assignment

The Casio fx-82MS calculator is a **bedmas** calculator.
If you load the correct sequence into your calculator you will get the correct answer, however one wrong button and your answer will be very wrong.

It is important therefore to understand the order of operations (**bedmas** rules) and to estimate a rough value to your calculation to check against your answer from your calculator. Calculating twice is also a good method of checking for keying errors. As an electrician checking and testing will become second nature as a requirement to safeguard lives, this philosophy starts here. Check your work.

Work out an estimate for each problem using **bedmas** rules, then solve twice on your calculator.

1) **$3 \times 4 + 8 =$**
estimate = calculation 1 = calculation 2 =

2) **$17 \div 8 + 5 =$**
estimate = calculation 1 = calculation 2 =

3) **$32 + 15 \div 4 =$**
estimate = calculation 1 = calculation 2 =

4) **$3 + (42 - 8) \div 2 =$**
estimate = calculation 1 = calculation 2 =

5) **$52 - 18 \div (27 \div 22) =$**
estimate = calculation 1 = calculation 2 =

6) **$(42 + 5) \times (22 - 32) =$**
estimate = calculation 1 = calculation 2 =

7) **$3 - 10 \times 0 \div 4 - 4 =$**
estimate = calculation 1 = calculation 2 =

8) **$\frac{\sqrt{16} - 2}{4} =$**
estimate = calculation 1 = calculation 2 =

9) **$(62 - 22 \div 4 + (2 + 24)) 2 =$**
estimate = calculation 1 =
calculation 2 =

10) **$(42 \div 7 + 18 - \sqrt{25} - 4) \times 0 \div 25674399876 =$**
estimate = calculation 1 =
calculation 2 =

Ratios

Ratios are a mathematical method of comparing the value of a shared quantity or to value the scale of what you work with to the real size.
Ratios are expressed in 4 forms and are all interchangable

1 : 4 or $\frac{1}{4}$ or 0.25 or 25% and all mean 1 share in four

ratio **fraction** **decimal** **percentage**

The ratio form reads as 1 part out of 4 parts
This is the same for the fractional form
The decimal is the simplified calculation of the fraction
The percentage scales the ratio to parts per hundred

The table below shows how the 3 forms relate to each other

percentage	0	10	20	30	40	50	60	70	80	90	100
decimal	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
fraction	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

We typically use ratios for scaling electrical layout drawings
Fractions and decimals are practically used to show progress completing tasks, allocating costs or material use.
Percentages bring different systems to a common scale for easier comparism

We will use examples on worksheets to practice ratio problem solving and converting between the different forms

Fractions homework assignment

Converting a decimal to a fraction on your *Casio fx82MS* is easy.
Write in the decimal number and press the = button to display the decimal on the answer (bottom) line.
Press the **A $^{b}/_{c}$** button and the fraction equivalent will be displayed.
Each time this button is pressed the answer will alternate between fractions and decimals.
This means that the same button can be used to convert decimals to fractions or to convert fractions to decimals.

The A $^{b}/_{c}$ format as an example.

Starting with a mixed number fraction One and a quarter Or $1\frac{1}{4}$

where the arrangement is A = 1 b = 1 c = 4

the display will read **1 ┘1 ┘ 4** meaning **One and a quarter**

To enter this into your calculator key the following sequence

1	A $^{b}/_{c}$	1	A $^{b}/_{c}$	4

Now practice converting the decimals to fractions and the fractions to decimals in the 10 questions below.

item	Initial format	Converted format
1	0.5	
2	0.25	
3	0.8	
4	0.62	
5	$\frac{7}{16}$	
6	$\frac{1}{8}$	
7	$\frac{8}{32}$	
8	$3\ ^{1}/_{2}$	
9	$1\ ^{5}/_{8}$	
10	$\frac{22}{7}$	

Percentage homework assignment

Your *Casio fx82MS* calculator can convert fractions to percentages by scaling the ratio of the two parts of the fraction to a per 100 ratio (per cent)
It needs to see both parts of the fraction to make a comparism.

For example,
Convert one quarter or $^{1}/_{4}$ to a percentage.

Key

1	÷	4	=

0.250 appears as the answer

Key

shift	%

25.00 appears as the answer. This is 25%

A decimal works by the same method, remembering the decimal is a ratio of 1.000.
Therefore we need to key,

0.25	÷	1	=	shift	%

25.00 appears as the answer. This is 25%. In essence we move the decimal point 2 places.

Try converting to percentages for these examples.

1) $^{3}/_{8}$ answer …………………….

2) $^{7}/_{16}$ answer …………………..…

3) 0.35 answer …………….………

4) 0.42 answer …………………..…….

5) A drum of cable has 163 metres of cable on it. 92 metres are then used to wire some lights. What percentage of the original drum is left?

answer ………..……

6) You are paid $420 for an electrical job. $219 goes to the wholesaler for the electrical fittings used on the job. What percentage of the total bill is yours?

answer …………….

1.1 INTRODUCTION

To understand many electrical principles, students should have a background in certain basic mechanical principles. This background in turn requires the adoption of a system of fundamental units and any derived units that may arise from the system. It is often necessary for students to have the ability to manipulate these units into other required forms by some mathematical process. In addition to knowing the usual basic mathematical processes, an understanding of graphs, graphical solution methods and trigonometry is also required for electrical studies. This chapter considers the units of measurement and mathematical processes that will aid the study of the technical subject matter in the remaining chapters.

1.2 BASE UNITS (SYSTÈME INTERNATIONALE)

There are six base units in the international metric system Système Internationale (SI), although there are many derived units of which the more relevant ones will be dealt with in section 1.3. An additional unit called a supplementary base unit is also relevant to the material in this book. It is the angle of rotation and is referred to as a plane angle.

For interest, brief definitions of these units are given here. More exact definitions are to be found in the Standards Australia publication AS.ISO 1000:1998 *The international system of units (SI) and its applications.*

Metre

A metre is equal in length to 1 650 763.73 wavelengths in a vacuum of the orange-red line spectrum for the isotope krypton-86.

Kilogram

The kilogram, first defined as the amount contained in 1000 millilitres of pure water at 0°C, is now the mass of a particular piece of platinum stored under special conditions in France.

Second

A second is an interval of time corresponding to 9 192 631 770 oscillations of a caesium-133 atom under specified conditions.

Table 1.1 • Base SI units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
length	l	metre	m
mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
temperature	T	kelvin	K
luminous intensity	I	candela	cd
angle of rotation (supplementary unit)	α	radian	rad

Ampere

An ampere is the current flowing in each of two parallel conductors of infinite length and negligible cross-section. When separated by a distance of 1 metre from each other in free space, 1 ampere produces between those conductors a force equal to 2×10^{-7} newton per metre length of conductor.

Kelvin

A kelvin is the unit of temperature equal to 1/273.16 of the triple-point temperature of water. The kelvin is used for absolute temperature measurements.

Candela

The candela is 1/60 of the lighting power emitted by 0.0001 square metre of a full radiator at the sea-level temperature of solidification of platinum.

Radian

A radian is the angle between two radii of a circle which mark off on the circumference an arc equal in length to the radius of the circle.

These base units, from which our other necessary units may be derived, are used to measure quantities that can vary considerably in magnitude. To avoid very large or small figures, prefixes representing multiples and submultiples are often used. The multiples and submultiples are listed in Table 1.5.

1.3 SI DERIVED UNITS

The six basic units are not sufficient to cater for all situations that arise in measurement. Derived units are used for all non-basic situations. Most derived units use the three basic units of length, mass and time in various combinations. The units used in this book can be subdivided into three groups: mechanical, electrical and magnetic, although it must be realised there are many more examples than those listed.

1.3.1 Mechanical

Newton

A newton is the force which, when applied to a mass of 1 kilogram, causes an acceleration of 1 metre per second per second.

Pascal

The pressure that occurs when a force of 1 newton is applied to an area of 1 square metre.

Energy and work

When a force of 1 newton is applied over a distance of 1 metre, the work done or energy expended is 1 joule.

Temperature

The temperature expressed in degrees Celsius (°C) is equal to the temperature expressed in kelvins (K) less 273.16. The intervals between °C and K are identical.

Angular velocity

In one revolution there are 2π radians. Angular velocity or speed of rotation is measured by revolutions per minute (r.p.m.) or by radians per second (rad/s).

Table 1.2 • Derived mechanical units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
force	*F*	newton	N
pressure	*P*	pascal	Pa
energy and work	*W*	joule	J
temperature	*t*	degree Celsius	°C
angular velocity[a]	ω	radians per second	rad/s
volume	*V*	cubic metres	m^3

[a] In accordance with AS/NZS 1046, angular velocity for practical cases can also be expressed as revolutions per minute and abbreviated as r/min.

Volume

The unit of volume is based on the unit of length and is the cubic metre. It is a large unit and for liquid measure the litre is used: 1000 litres = 1 cubic metre. The litre in turn has its submultiples such as the millilitre; that is, 1000 mL = 1 litre.

1.3.2 Electrical

Watt

A watt unit is the power used when energy is expended at the rate of 1 joule per second.

Coulomb

A coulomb is the quantity of electric charge transferred each second by a current of 1 ampere (nominally equal to 6.24×10^{18} electrons).

Hertz

A hertz is the number of periodic oscillations per second (frequency).

Volt

A volt is the potential difference existing between two points on a conductor carrying a current of 1 ampere when the power dissipated is 1 watt.

Farad

A farad is the capacity that exists between two plates of a capacitor if the transfer of 1 coulomb from one plate to the other creates a potential difference of 1 volt.

Table 1.3 • Derived electrical units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
power	*P*	watt	W
charge	*Q*	coulomb	C
frequency	*f*	hertz	Hz
potential	*V*	volt	V
capacity	*C*	farad	F

1.3.3 Magnetic

Weber

A weber was once a unit of 10^8 lines of force. Now it is the magnetic flux linking one turn that produces 1 volt if reduced to zero at a uniform rate in 1 second.

Tesla

A tesla is a magnetic flux of 1 weber per square metre.

Henry

An inductance has a value of 1 henry when an electromotive force (e.m.f.) of 1 volt is produced by a current changing uniformly at a rate of 1 ampere per second.

Table 1.4 • Derived magnetic units

Quantities		Units	
Physical quantity	Quantity symbol	Unit name	Unit symbol
flux	Φ	weber	Wb
flux density	*B*	tesla	T
inductance	*L*	henry	H

1.3.4 Multiples and submultiples

In practical cases some SI values are inconveniently large or small. In order to choose values that are convenient to handle, multiples or submultiples are used. For example, if the resistance of an electrical installation is measured at 15 000 000 ohms, it is more convenient to refer to this value as 15 megohms; that is, 15 units, each consisting of one million ohms (see Table 1.5). Similarly, it is easier to refer to the output of a power station as 125 megawatts (125 MW) than 125 000 000 watts. The unit of capacity is the farad. This is a large unit for most applications so it is usual to refer to capacity in microfarads or picofarads.

Table 1.5 • SI multiples and submultiples

Grouping	Notation	Symbol	Example
tera	10^{12}	T	1 THz = 1 000 000 000 000 Hz
giga	10^9	G	1 GHz = 1 000 000 000 Hz
mega	10^6	M	1 MHz = 1 000 000 Hz
kilo	10^3	k	1 kHz = 1 000 Hz
Unit	Notation	Symbol	Example
milli	10^{-3}	m	1 mH = 0.001 H
micro	10^{-6}	μ	1 μH = 0.000 001 H
nano	10^{-9}	n	1 nH = 0.000 000 001 H
pico	10^{-12}	p	1 pH = 0.000 000 000 001 H

Extract from Jenneson " electrical principles for the electrical trades" 5th edition 2007

Scientific Notation

A number is said to be in scientific notation when it is written as a number between 1 and 10, times a power of 10.

e.g 634.8 = 6.348×10^{2} and 0.00234 = 2.34×10^{-3}

Note: When starting with a basic number,
if the decimal point is moved **left,** then the power of ten is **positive**,
but if the decimal point is moved **right**, then the power of ten is **negative**.

For **engineering calculations**, powers of 10 are in multiplies of 3 as these align with **metric prefixes**:

e.g. 10^{12}, 10^{9}, 10^{6}, 10^{3}, 10^{0}, 10^{-3}, 10^{-6}, 10^{-9}, 10^{-12}.

Most engineering type calculators have an ENG key that does the above for you i.e. You do not have to put in "x10". You only press ENG then the power figure (default +ve) or –ve then the power figure if it's a –ve power.
You must **not** use the 10^{x} key – It is likely to give you incorrect answers.

Metric prefixes.

These are Greek letters written after the number and before the unit symbol to indicate a multiplier or divider. i.e these letters are used instead of " 10 to the power of " but represent the same thing. The letters have "names" and we use them everyday
e.g. kilogram, millimetre, Mega-centre.

Pico	**ρ**	$\frac{1}{1{,}000{,}000{,}000{,}000}$	1×10^{-12}
Nano	**η**	$\frac{1}{1{,}000{,}000{,}000}$	1×10^{-9}
Micro	**μ**	$\frac{1}{1{,}000{,}000}$	1×10^{-6}
Milli	**m**	$\frac{1}{1{,}000}$	1×10^{-3}
(Basic unit	n.a.	1	1×10^{0})
Kilo	**k**	1 x 1,000	1×10^{3}
Mega	**M**	1 x 1,000,000	1×10^{6}
Giga	**G**	1 x 1,000,000,000	1×10^{9}
Tera	**T**	1 x 1,000,000,000,000	1×10^{12}

SI units quiz **Name**

1) Circle the biggest Mega milli micro
2) Circle the smallest Mega milli micro
3) Circle the base unit Kilogram gram milligram
4) Circle the smallest Kilogram gram milligram
5) Circle the base unit Hour minute second
6) Circle for 10^6 nano Mega Tera
7) Circle for **μ** milli micro nano
8) Circle for 0.001sec microsec millisec nanosec
9) Circle for 1 metre 10^6 micrometre 10^{-6} kilometre
10) Circle the base unit centimetre millimetre kilometre

10 marks

Fill in the missing information

10^{-12}			10^{-3}	10^0		10^6		10^{12}
		μ		Base			G	
	nano			--------	kilo			

16 marks

Number format

Take any number in **ORDINARY NOTATION**

e.g. 225,000 or 0.00056

(always put a 0 in front of a decimal point when it starts the number.)

You can turn ordinary numbers into **STANDARD FORM** by moving the decimal point either left or right by any number and using the '10 $^{\text{to the power of}}$' system:

	225×10^3	or	0.56×10^{-3}
or	22.5×10^4	or	5.6×10^{-4}
or	2250×10^2	or	56×10^{-5}

To express the above in **SCIENTIFIC NOTATION** then the number must be written as a number between 1 and < (less than) 10, times a power of 10. For the above, they could only be written as:

2.25×10^5 5.6×10^{-4} (Mode : SCI on most calculators)

When we use **ENGINEERING NOTATION** then the 'powers of 10 ' can only be written in multiples of +3 or –3. The above Scientific Notation would then have to be written as:

225×10^3 560×10^{-6} (ENG key on calculators)

(It is preferred to not start any figure with a nought point something (0.---)

E.g. **Not** 0.225×10^6 or 0.56×10^{-3}

Alternatively, in **Engineering** we can replace the ' 10 $^{\text{to the power of}}$ ' with a **Multiple** that represents 'x 10 $^{\text{plus ---}}$ ',
or with a **Sub-multiple** that represents 'x 10 $^{\text{minus ----}}$ '

These are always in multiples of $^{+3}$ or $^{-3}$.
For example, if the above were Amps:

225 kA ($k = 10^3$) 560 μA ($\mu = 10^{-6}$)

Again, it is best not to start the number with 0.--- , however, if you gave an answer starting with 0.--- then we would not mark it wrong.

In the Electrical and Electronic Industries, we use <u>ENGINEERING NOTATION.</u>

(either in ' 10 $^{\text{to the power of multiples of +3 or -3}}$ ' or with Multiples / sub-multiples)

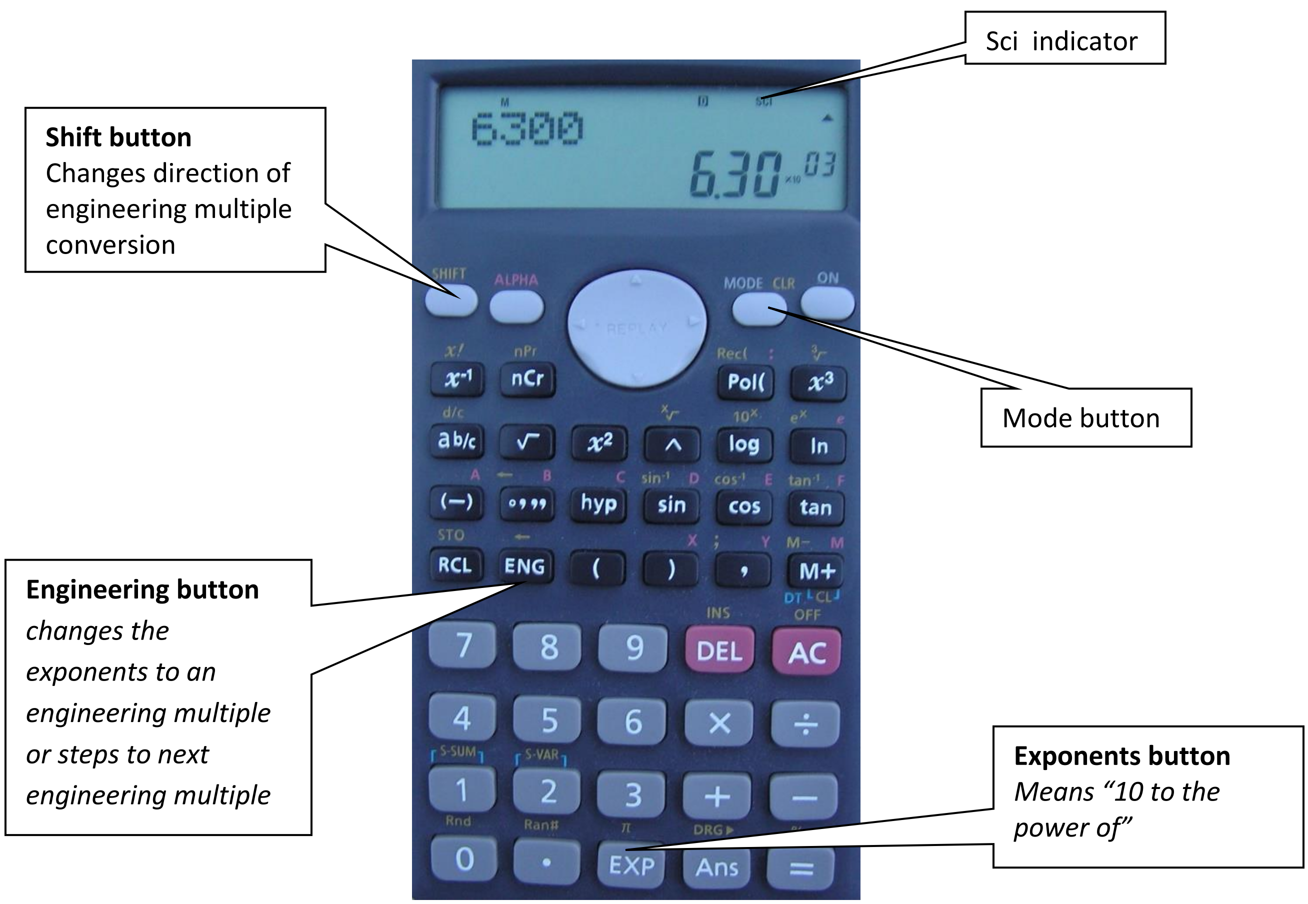
Engineering units worksheet

Calculator use

The casio fx-82MS can be set up for scientific notation by pressing the mode button 3 times. Select option 2 and then 3.
Scientific format should now appear on the screen and a small **sci** icon at the top of the screen. We are set up for engineering conversions now.

Obviously we need to know the scaling or multiples of the engineering values shown in line format below. We are constantly referring to this conversion chart and need to memorize it.

ρ	n	μ	m	unit	k	M	G	T
10^{-12}	10^{-9}	10^{-6}	10^{-3}	1	10^{3}	10^{6}	10^{9}	10^{12}
pico	nano	micro	milli		kilo	mega	giga	tera



Practically we would not ask for 3000mls of milk at the supermarket, instead we would say 3 litres. It means the same quantity but not appropriate.

A 0.006 metre drill bit sounds a bit silly too, but 6mm(the same size) is easier to understand.

Example A Converting 220000 volts to an appropriate engineering multiple would be easier to communicate values to another tradesman
Using the calculator check the sci indicator is on and write **220000 =**
2.20 x 10^{05} appears at the bottom part of the display. *This is now in scientific form*
Press the **ENG (engineering) button**.
The answer displayed changes to 220 x 10^{03} (*10^3 from the chart means kilo)*
So the appropriate value unit is 220 kilo and the accompanying unit is volts
220 kilovolts or kV
(Note the 220 is a number between 0 and 1000)

Example B Convert 120000ρF to microfarads.
From the chart we see ρ is 10^{-12}
Write 120000 exp –12 =
1.20 x 10^{-07} appears at the bottom part of the display.
Microfarads are 10^{-6} Farads so we want to see this unit displayed.
Press ENG
120 x 10^{-09} appears now rather than 10^{-6}
Press SHIFT then ENG
0.12 x 10^{-6} appears now which is 0.12 microfarads

Practice these methods to complete the chart below, paying attention to units.

Ordinary number	**Scientific notation**	**Engineering notation**	**Appropriate form**
220000v	2.2 x 10^5	220 x 10^3	220kV
0.0003Ω			
	5.67 x 10^{-4}		
			17kA
65000000000W			
	2.2 x 10^{-7}		
33000000V		33 x 10^6	
			92µA
	1.7 x 10^5		
0.000056F			
		2 x 10^{-9}	
			100TBytes
	4 x 10^2		
476A			
0.0000000009C			
	5 x 10^{12}		

Engineering multiples & sub-multiples worksheet 1

1. Convert to the given multiple or sub-multiple:

50,000 Ω	=	**50**	kΩ	0.000 05 A	=	**50**	μA
2435 V	=		kV	0.002 5 A	=		mA
500,000,000 Ω	=		MΩ	0.002 W	=		mW
4,500 A	=		kA	2 W	=		mW
12,340,000,000,000 Wh	=		TWh	0.000 14 Ω	=		μΩ
270,000 W	=		MW	0.056 V	=		mV
25,450 V	=		kV	0.000 000 000 04 F	=		pF
265.5 A	=		kA	0.000 000 000 04 F	=		nF
0.000 023 4 J	=		μJ	45,000 V	=		kV

2. Convert the following to base units:

20 MΩ	=	**20,000,000**	Ω	200 mA	=	**0.2**	A
1500 kV	=		V	1200 mA	=		A
50 kA	=		A	0.54 mA	=		A
1.6 kV	=		V	12.6 mH	=		H
0.56 MΩ	=		Ω	500,000 nF	=		F
0.2 kV	=		V	50 μF	=		F
0.000 5 GW	=		W	85000 mA	=		A
0.000 005 TWh	=		Wh	85,000,000,000 pH	=		H
0.016 kV	=		V	25.64 μH	=		H

3. Convert to the given multiple or sub-multiple:

20,000 mΩ	=	**0.02**	kΩ	250 nF	=	**250,000**	pF
345 kV	=		MV	60,000 kΩ	=		MΩ
0.789 mH	=		μH	0.045 μF	=		mF
57,000,000 mV	=		kV	65,000 kΩ	=		mΩ
107 pF	=		nF	0.079 nF	=		mF
0.000 05 kV	=		mV	0.026 nA	=		pA

Engineering notation, multiples & sub-multiples worksheet 2

1. **Express the following in appropriate Engineering Notation (in ‘ x 10 to the power of ± 3 ’ format).**

45,000,000 Wh	=	**45×10^6**	Wh	0.000 05 A	=	A
0.025 6 A	=		A	2000 V	=	V
65,000 J	=		J	45,340,000,000 J	=	J
27,200 V	=		V	0.000 000 005 78 F	=	F

2. **Complete the following table, use appropriate values:**

	Ordinary Notation.	Multiple / Sub-multiple.	Engineering Notation.
e.g ``	**25,000 V**	**25 kV**	**25×10^3 V**
	0.003 54 Ω		
	A	27 mA	
			500×10^{-6} C
	µF	15 nF	
	GWh		87.2×10^6 Wh
	4,500 A		
	0.005 mA		
	mH		256×10^{-9} H
	0.00814 nF		
	VA	25.87 kVA	
	7,500,000 J		
	V		56.23×10^3 V
	mF		56×10^{-6} F

Squares and square roots

The **square** of a number is the number multiplied by itself.

For example - The square of 3 is **9** (because 3 x 3 = 9)

4 squared means 4 times itself and is written mathematically as $\mathbf{4^2}$

To write this into your calculator you enter **4** then press the x^2 button

The **square root** of a number is a number that when multiplied by itself gives the original number.

For example - The square root of 9 is 3 (because 9 = 3 x 3)

The square root of 9 is written mathematically as $\sqrt{9}$

The square root sign covers the numbers affected acting as if it was a bracket.

$$\sqrt{9+16}$$

The square root function starts

And finishes here

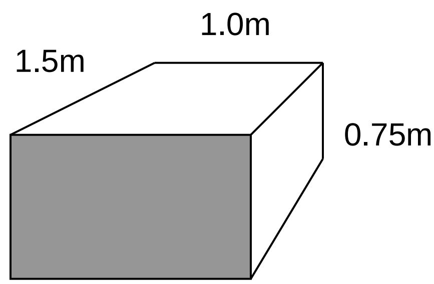
This reads as the square root of (9 + 16) , which equals 5

Not the square root of 9 and then plus 16 , which equals 19

Area and Volume

Area is 2 dimensional. The surface or plane, of or on or around an object.
We do not go inside an object when considering the areas of surfaces.
Area is given in units of **m^2**

Volume is 3 dimensional. This is the amount of room inside an object.
Volume is given in units of **m^3**



This box has 6 surfaces. The **area** of the shaded surface is calculated by multiplying the height by the width.

1.0m x 0.75m = 0.75**m^2**

To find out the **total surface area** of this object we would need to calculate the surface of each of the 6 sides and add them together. The answer will be in m^2 still as it is still a measure of area.

1.0m x 0.75m = 0.750m^2
1.0m x 1.5 m = 1.500m^2
1.5m x 0.75m = 1.125m^2
3.375m^2
x2
6.750**m^2**

To find out the volume or amount of room inside the box we simply multiply the 3 dimensions together.

1.0m x 0.75m x 1.5m = 1.125**m^3**

Remember area is always m^2 and volume is m^3

Area formulae

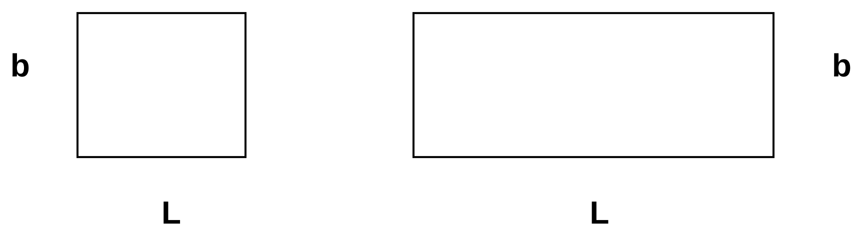
The area of a **square** and an **oblong / rectangle** are both covered by the formula

Length x breadth = area

metres x metres = metres^2 or square metres

The units we traditionally use in engineering as 10^3 exponents are mm^2, m^2, and km^2.

The formula we use for the area of these shapes is **A = L x b**



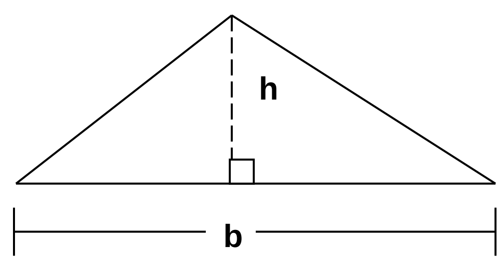
The area of a **triangle** is calculated by multiplying the perpendicular height of a triangle by half its base length.

$^1/_2$ base x height = area

metres x metres = metres^2

The formula for the area of a triangle is

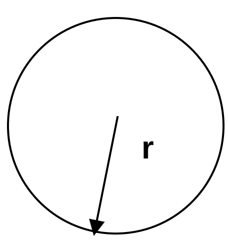
A = $^1/_2$ b x h

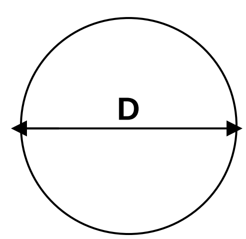


The area of a **circle** is calculated using its radius where the radius squared is multiplied by π. This is given in metres^2

The **circumference** or the distance around the outside is calculated by multiplying π by the diameter. This is given in lineal metres.

area = $\pi \times r^2$





circumference = $\pi \times D$

Area and Volume homework worksheet

1) Calculate the area of a circle with a radius of 20mm

working answer ……………

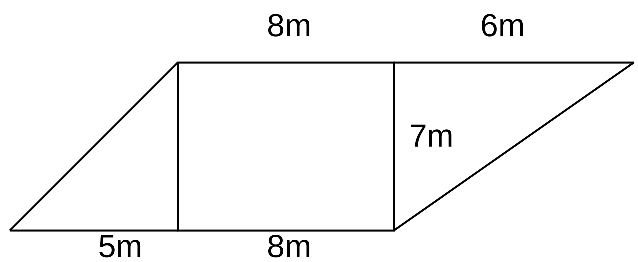
2) Calculate the volume of water in a hot water cylinder with a 200mm diameter circular top and a height of 1.5m, in both m^3 and mm^3

working

answer…………………..….m^3
answer ………………..….mm^3

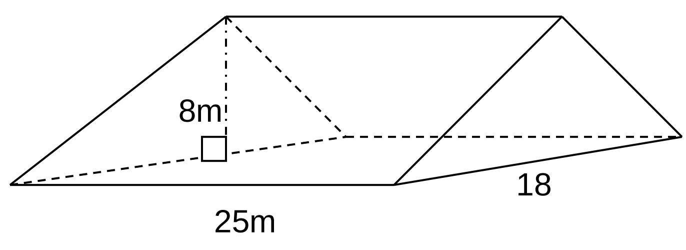
3) Find the area of this shape

Working.



answer ……..m^2

4) How much volume of air is inside this roof space. working



answer ………m^3

5) A 100m copper cable has 4 cores measuring 8mm diameter.
When the copper is melted down and poured into a 500mm diameter container, how high will it be filled. *Use overleaf for extra working*

answer ……….m

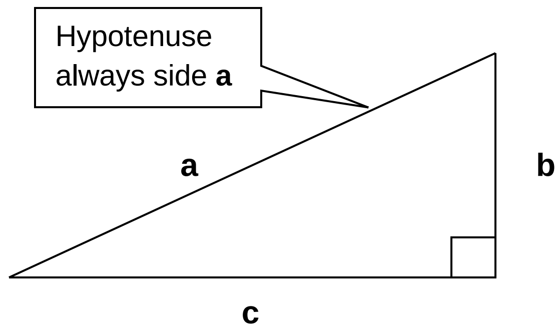
Trigonometry - Pythagoras

Pythagoras was a Greek mathematician credited with discovering the relationship between the lengths of the 3 sides on any right angled triangle.
This relationship is called **Pythagoras' theorem**

Remember this applies only to **right angled** triangles

The longest side of a right angled triangle, which is opposite to the right angle is called the **hypotenuse**

Pythagoras' theorem states,



In any right angled triangle,
the square of the hypotenuse
is equal to the sum of the squares
of the other 2 sides

and in equation form

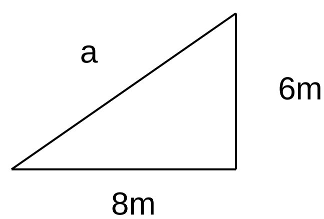
$\mathbf{a^2 = b^2 + c^2}$

example 1) $a^2 = b^2 + c^2$

$a^2 = 6^2 + 8^2$

$a^2 = 36 + 64$

$a^2 = 100$



we now square root both sides

$a = \sqrt{100}$

$\underline{a = 10m}$

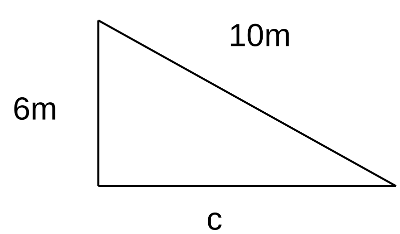
example 2) $a^2 = b^2 + c^2$

$100^2 = 6^2 + c^2$

$100 = 36 + c^2$

$100 - 36 = c^2$

$64 = c^2$



we now square root both sides

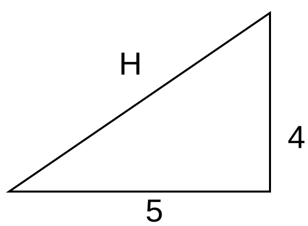
$\sqrt{64} = c$

$\underline{c\ =\ \ }$

Pythagoras worksheet 1

Calculate the length of each unknown side of the following right-angled triangles.
Show your line by line working for each question.
Calculate to 2 decimal places

1)



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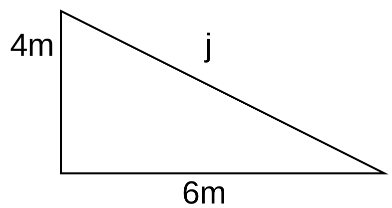

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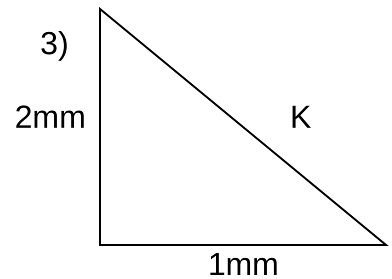
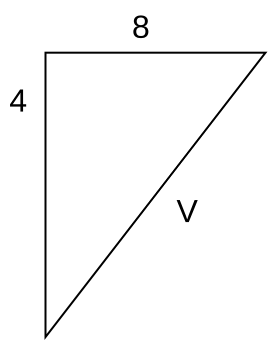
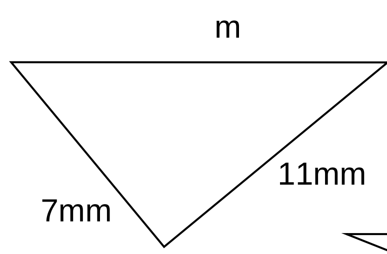


4)



5)



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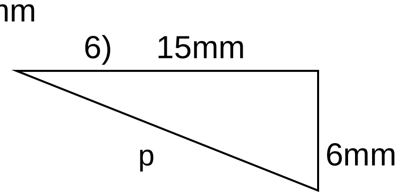

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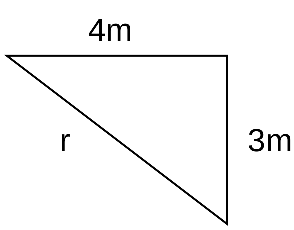

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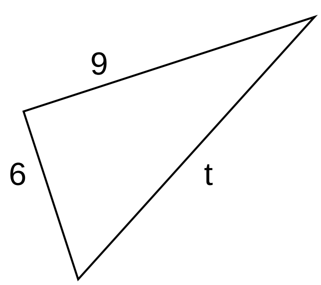

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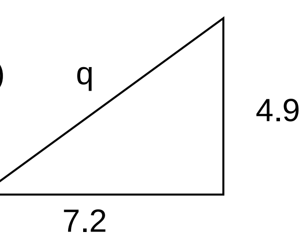
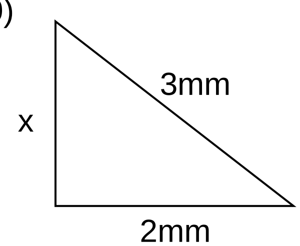


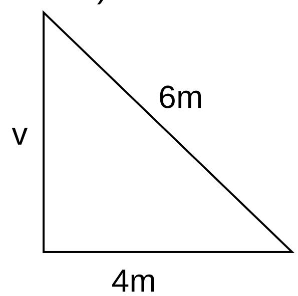
Pythagoras worksheet 2

Calculate the length of each unknown side of the following right-angled triangles.
Show your line by line working for each question.
Calculate to 2 decimal places

10)



11)



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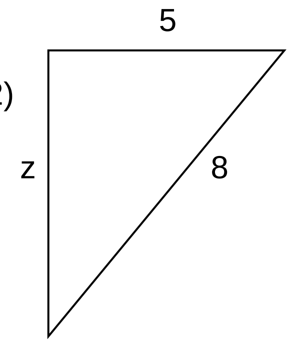

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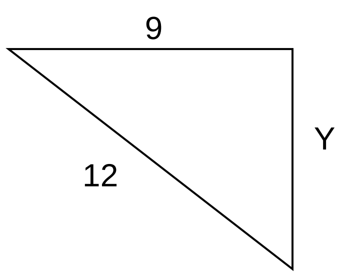
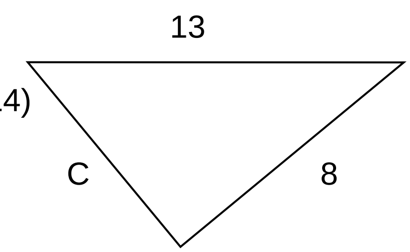
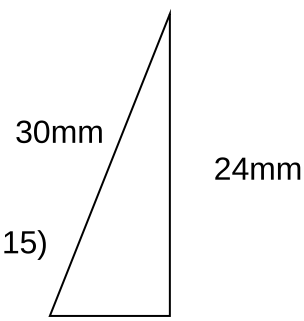


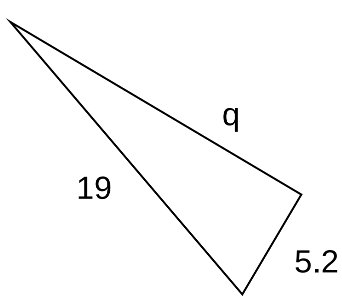
14)



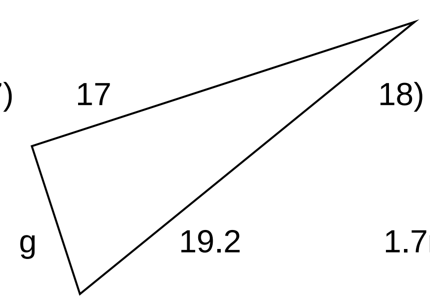
15)



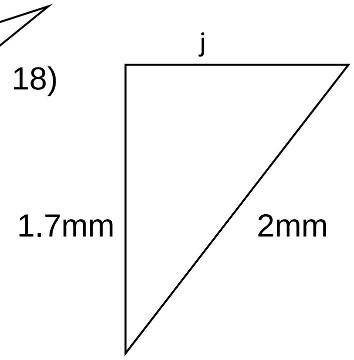
16)



17)



18)



Pythagoras worksheet 3

Pythagoras' theorem can be used to solve engineering problems.
In the 2 problems below, draw a sketch to illustrate the scenario, adding the values to the sketch.
Lastly calculate the solution showing line by line working to 2 decimal places.

1) You have a 3.5m ladder. It is to be placed against a wall with a safe distance out from that wall at the ladder foot of 0.8m. How far up the wall will it reach?

2) A rectangular wall needs a diagonal brace to be fitted from the right bottom corner to the left top corner. The wall is 3.2m high and 6.8m long.
Will a 7.3m brace be long enough?

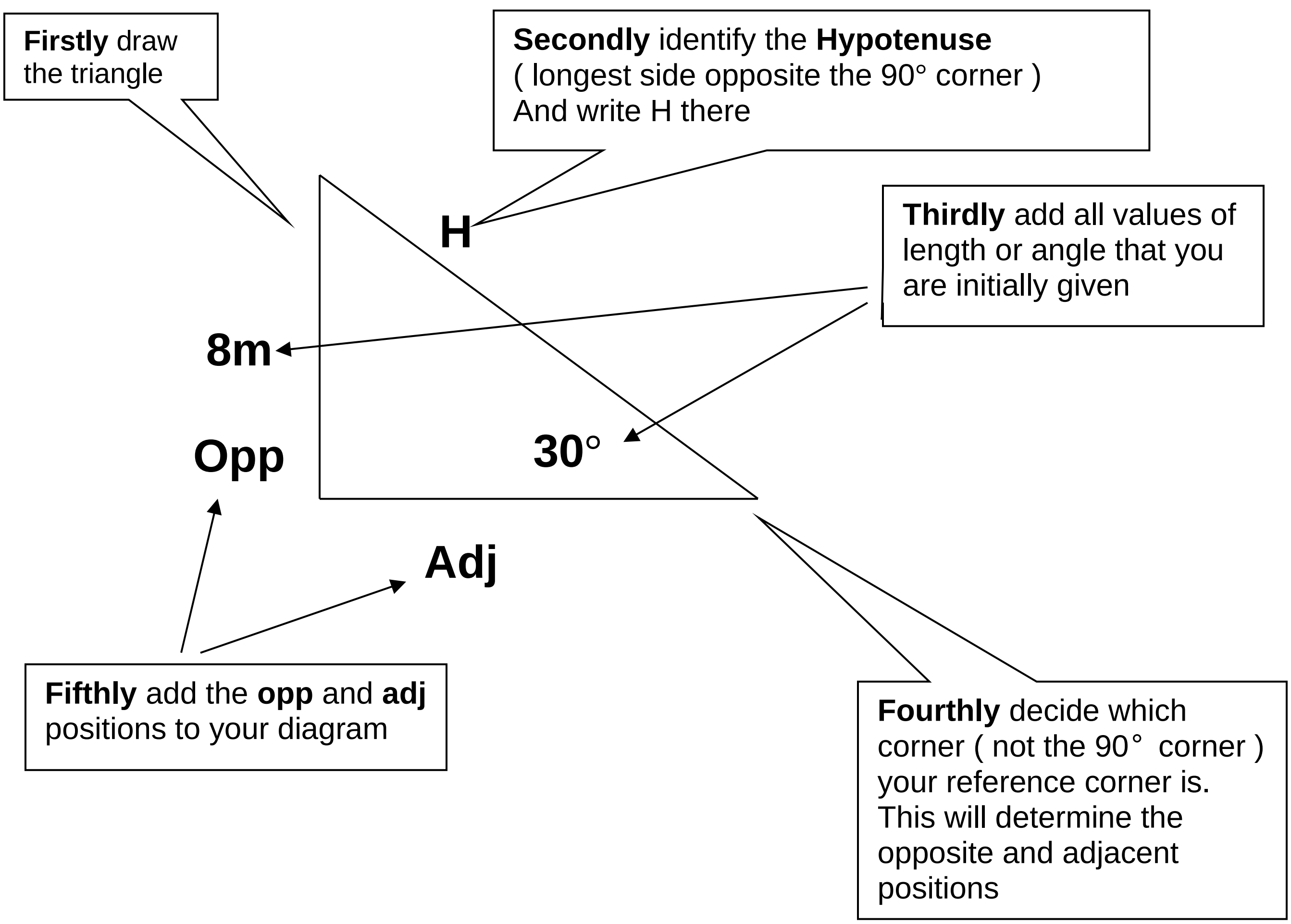
WS1&2 / 1) 6.40, 2) 7.21m, 3) 2.24mm, 4) 8.94, 5) 13.04mm 6) 16.16mm,
7) 5m, 8) 10.82, 9) 8.71, 10) 2.24mm, 11) 4.47m, 12) 6.24, 13) 7.94
14) 10.25, 15) 18mm, 16) 18.27, 17) 8.92, 18) 1.05mm
WS3 1) 3.4m, 2) 7.5m

Trigonometry - SOHCAHTOA

Pythagoras allows us to find the 3rd sides length of a right angled triangle if we know the other 2 lengths.

If **angles** are involved we need to use the trigonometric ratios (SOHCAHTOA)
We can use these ratios to find the remaining lengths and angles if we know 2 values of any length or angle.

We need to have a common approach to solving these various problems.



We are now ready to start using the **SOHCAHTOA** acronym

SOHCAHTOA is an acronym that helps you to remember the 3 following trigonometric ratios.

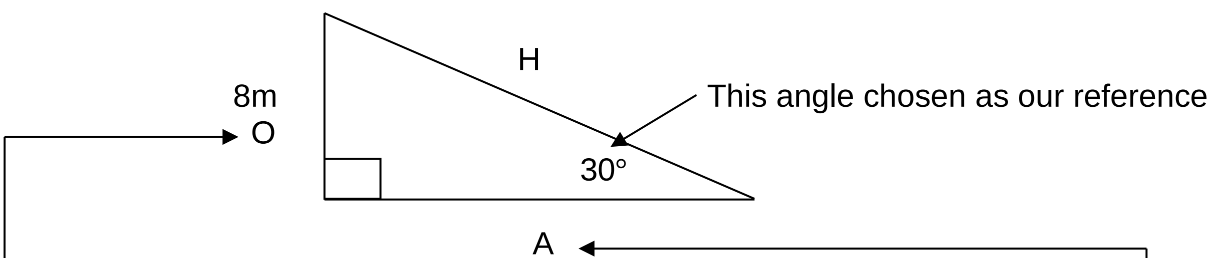
SOHCAHTOA splits into 3 ratios

Sine of this angle = $\dfrac{\text{the length of the side Opposite to this angle}}{\text{the length of the Hypotenuse}}$ **SOH } S =** $\dfrac{\mathbf{O}}{\mathbf{H}}$

Cosine of this angle = $\dfrac{\text{the length of the side Adjacent to this angle}}{\text{the length of the Hypotenuse}}$ **CAH } C =** $\dfrac{\mathbf{A}}{\mathbf{H}}$

Tangent of this angle = $\dfrac{\text{the length of the side Opposite to this angle}}{\text{the length of the side Adjacent to this angle}}$ **TOA } T =** $\dfrac{\mathbf{O}}{\mathbf{A}}$

We will work through the example given above now that we have the basic rules



1) The longest side opposite the right angle is H
2) The 8m side is opposite the chosen angle so becomes the O
3) The remaining side, which is not the H, becomes the A as it is next to the chosen angle
4) We know the value of our chosen angle **30**°
5) We know the value of the Opposite **8m**

6) **SOH** **CAH** **TOA**

SOH:

$\text{Sin } 30° = \dfrac{8m}{\mathbf{H}}$

$\mathbf{H} = \dfrac{8m}{\text{Sin } 30}$

$\mathbf{H} = \dfrac{8m}{0.5}$

H = 16m

CAH:

A & H **both** unknown

TOA:

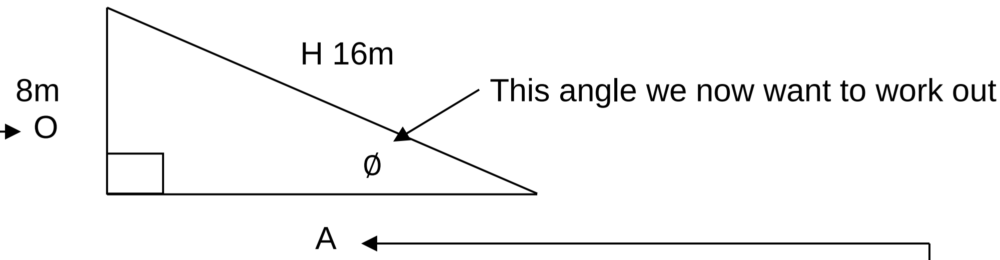
$\text{tangent } 30° = \dfrac{8m}{\mathbf{A}}$

$\mathbf{A} = \dfrac{8m}{\tan 30}$

$\mathbf{A} = \dfrac{8m}{0.578}$

A = 13.856

We will now work through the same example but start with **2 lengths** and **calculate the angle**



1) The longest side opposite the right angle is H and is **16m**
2) The 8m side is opposite the chosen angle Ø so becomes the O
3) The remaining side, which is not the H, becomes the A as it is next to the chosen angle
4) We will calculate the value of our chosen angle Ø
5) We know the value of the Opposite **8m**

6) **SOH** | **CAH** | **TOA**

$\text{Sin } \emptyset = \dfrac{8m}{16m}$ | A & Ø **both** unknown | A & Ø **both** unknown

$$16m = \frac{8m}{\text{Sin } \emptyset}$$

$$16m \times \text{Sin } \emptyset = 8m$$

$$\text{Sin } \emptyset = \frac{8m}{16m}$$

$$\text{Sin } \emptyset = 0.5$$

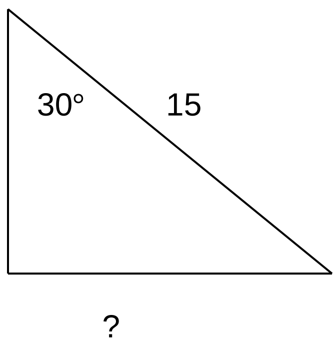
$\emptyset = \text{Sin}^{-1}\ 0.5$ we type shift / sin / 0.5 / = on the calculator

$\underline{\emptyset = 30^\circ}$

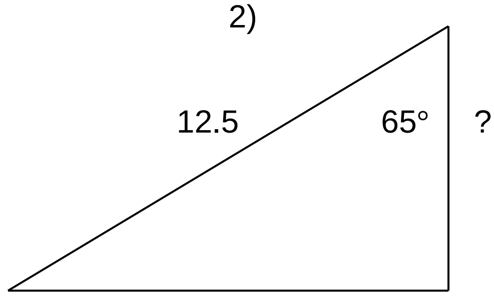
SOHCAHTOA worksheet 1

Calculate the length of the **? unknown side** of the following right-angled triangles.
Show your line by line working for each question. Calculate to 2 decimal places.

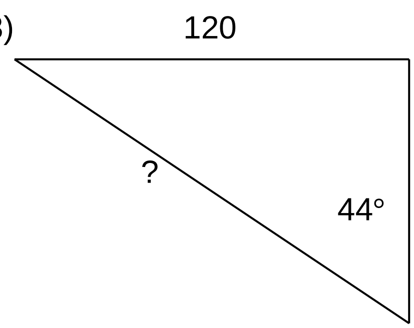
1)



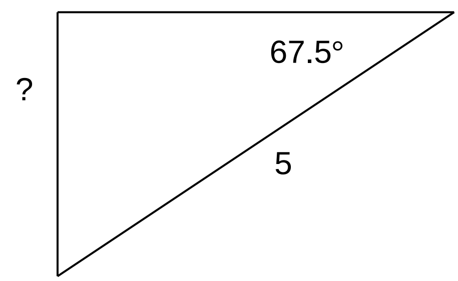
2)



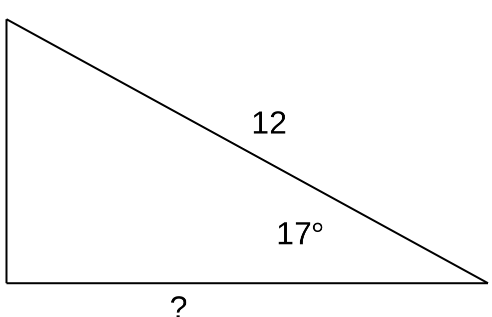
3)



4)



5)

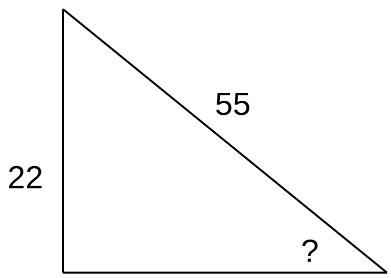


1) 7.5 2) 5.28 3) 172.75 4) 4.62 5) 11.48

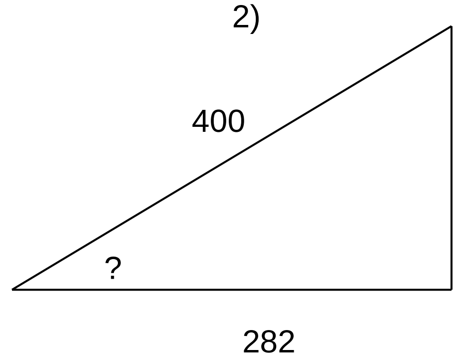
SOHCAHTOA worksheet 2

Calculate the length of the **? unknown angle** of the following right-angled triangles.
Show your line by line working for each question. Calculate to 2 decimal places.

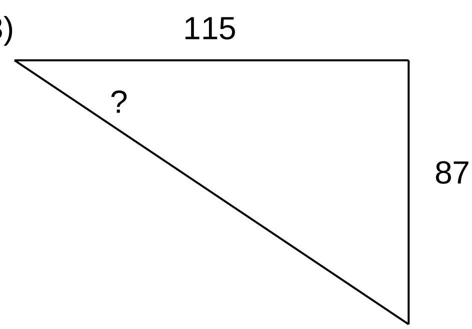
1)



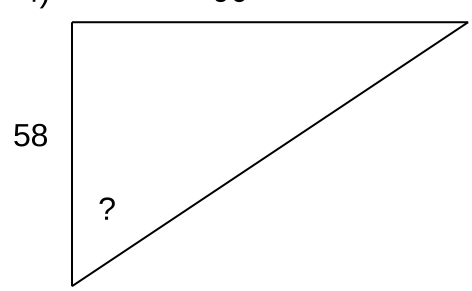
2)



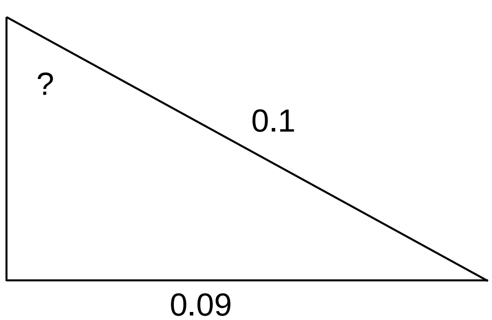
3)



4)



5)



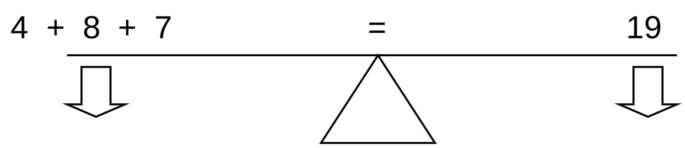
1) 23.58° 2) 45.17° 3) 37.12° 4) 57.2° 5) 64.16°

Equations

An equation is a mathematical expression of information.
Numbers are used to convey known values.
Letters are used to substitute for unknown values.
We can change the letters to numbers as their values become known to us.

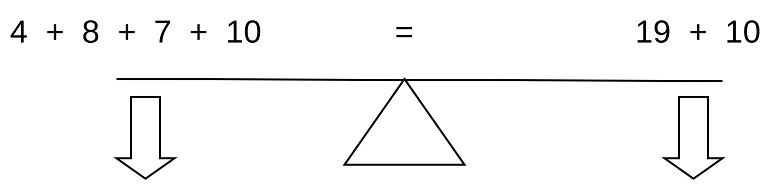
If we know all the values as numbers except for one unknown letter, we can solve its value as a number.

All equations have 2 sides separated by an equals sign.
This works the same as a seesaw or balance scales where the pivot is the equals sign.



To maintain the balance we can see that a principal rule must be,

1) ***Whatever changes are made to one side of the equals sign, the same must also occur on the other side***



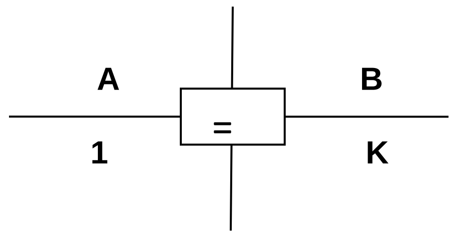
"To solve an equation" means to rearrange a formula to end up with all the known values on one side of the equals sign, and one unknown, a letter (the subject), on the other.
Rearranging a formula is also call **transposition.**

We need to rearrange the formula to strict rules and in a certain order.
This is usually the tricky part.
In essence the order is in reverse of BEDMAS.

Subtraction or addition,
Division or multiplication,
Exponents
Brackets

A format to put your original formula into, which may help you visualise the division or multiplication rearranging, is shown below.

$$A = \frac{B}{K}$$



This gives a definite top line and bottom line to the left and right of the equals sign, or 4 quadrants. The horizontal line is the divide line. If there is no division in the equation then it can be helpful to place a 1 in the bottom quadrant to see that a value does exist here.

We can now start with the next set of rules.

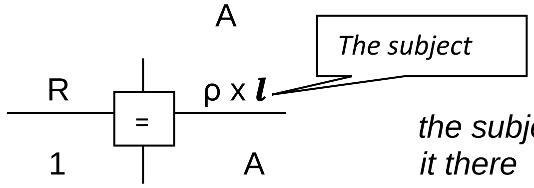
- **2) Get the subject onto a top line**
- **3) Isolate the subject by shifting all the other information to the other side of the equals sign.**
- **4) When the information crosses the equals sign it will move,**
- **a) from the top to the bottom, or**
- **b) from the bottom to the top**

If we now look at a simple example $R = \frac{\rho \times l}{A}$ (an actual equation that we use.)

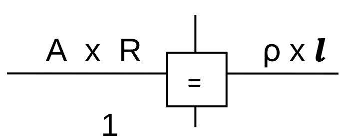
We want to find the length of a conductor ( ) and we know the other values.
We will therefore solve for .

$$R = \frac{\rho \times l}{A}$$

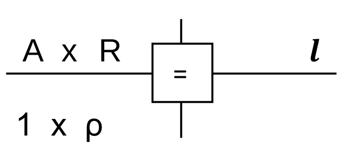
Convert the view to 4 quadrants



the subject is on a top line so we leave it there



The A has crossed to the diagonally opposite quadrant
Only ρ needs to move over to isolate l



The l has crossed to the diagonally opposite quadrant

$$l = \frac{R\,A}{\rho}$$

Finally shown as a transposed equation

When we transpose equations with the other BEDMAS operations involved the rules become more complicated, and are best understood on an example by example basis

Example 1 $5 + 3x = 7$ *We want to leave the x where it is and shift everything else to the other side of the equals sign.*

$$\mathbf{-5} + 5 + 3x = 7 - \mathbf{5}$$ *5 has been deducted from **both** sides and the +5 and -5 cancel each other out leaving*

$$3x = 7 - 5$$ ***both** sides are now divided by 3*

$$\frac{3x}{\mathbf{3}} = \frac{7-5}{\mathbf{3}}$$ *the 3 top and bottom of the x cancel each* other

$$x = \frac{7-5}{3}$$

$$x = 0.667$$

Example 2

If we now construct the same equation with all the values unknown, following the steps above

$$A + Bx = C$$

$$A - A + Bx = C - A$$ *$-A$ from both sides*

$$Bx = C - A$$ *simplify (removes both A's from LHS)*

$$\frac{Bx}{B} = \frac{C-A}{B}$$ *divide both sides by B*

$$x = \frac{C-A}{B}$$ *simplify (LHS B's both cancel out)*

Transposition worksheet 1

Re-arrange the given equation (i.e. transpose the formulae) to solve for the figure that is to the right of the equation.

1. $I = \dfrac{V}{R}$ $R =$

2. $P = V \times I$ $I =$

3 $I = \dfrac{V}{R}$ $V =$

4. $W = \dfrac{Z}{V}$ $Z =$

Now try the same as above with some Greek Letters α say Alpha, β say Beta, ε say Epsilon, ϕ say Phi, π say Pi, μ say Mu.

5. $\beta = \mu \times \alpha$ $\alpha =$

6. $\varepsilon = \dfrac{\phi}{\pi}$ $\pi =$

7. $\mu = \dfrac{\beta}{\alpha}$ $\beta =$

8. $Cos\,\phi = \dfrac{A}{H}$ A =

9. $X_L = 2\pi f L$ $f =$

10. $X_C = \dfrac{1}{2\pi f C}$ $C =$

11. $\beta = \mu_0 \times \mu_r \times H$ $H =$

12. $Q = \frac{2\pi f_0 L}{R}$ $R =$

13. $Q = m \times c \times \Delta t$ $\Delta t =$

(the Delta and the t are combined in one symbol)

14. $W = \frac{1}{2} C V^2$ $C =$

15. $L = \frac{\mu_0 \mu_r N^2 A}{l}$ $N =$

16. $R_1 = R_0 (1 + \alpha_0 t_1)$ $R_0 =$

17. $f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$ $L =$

18. $\frac{R_2}{R_1} = \frac{1 + \alpha_0 t_2}{1 + \alpha_0 t_1}$ $t_2 =$

Answers

1) $R = \frac{V}{I}$

2) $I = \frac{P}{V}$

8) $V = I \times R$

9) $Z = W \times V$

10) $\alpha = \frac{\beta}{\mu}$

11) $\pi = \frac{\phi}{\varepsilon}$

12) $\beta = \alpha \times \mu$

13) $A = H \times Cos\phi$

14) $f = \frac{X_L}{2\pi L}$

10) $C = \frac{1}{2\pi f X_C}$

11) $H = \frac{\beta}{\mu_0 \times \mu_r}$

12) $R = \frac{2\pi f_0 L}{Q}$

13) $\Delta t = \frac{Q}{m \times c}$

14) $C = \frac{2W}{V^2}$

15) $N = \sqrt{\frac{l\,L}{\mu_0 \mu_r A}}$

16) $R_0 = \frac{R_1}{1 + \alpha_0 t_1}$

17) $L = \frac{1}{C(f_0 2\pi)^2}$

18) $t_2 = \frac{\left[\frac{R_2(1 + \alpha_0 t_1)}{R_1} - 1\right]}{\alpha_0}$