MECHANICS – PART 1

A Companion to Book "D"

INTRODUCTION

Mechanics deals with systems that have moving parts.

Motors and generators convert electrical energy to mechanical energy types and vice versa.

Mechanics helps us answer questions like:

How powerful will a motor need to be?

How much will it cost to run some equipment?

Is a crane capable of lifting a certain load?

Is a generator of sufficient size to run a motor at full power?

MECHANICS TERMS

Some important terms and concepts in mechanics:

- **Speed** – symbol *v* - rate of change of distance travelled over time – unit is metres per second ($m.s^{-1}$). Speed is also related to **velocity**
- **Acceleration** – symbol *a* – the rate of change of speed or velocity over time – unit is metres per second squared ($m.s^{-2}$)
- **Mass** – symbol *m* - amount of matter in an object – unit is kilogram (kg)
- **Force** – symbol *F* - something that will accelerate an object if it is not balanced – unit is newton (N)
- **Work** – symbol *W* - Energy transferred from one system (e.g. electrical) to another (mechanical) – unit is joule (J) or newton metre (Nm)

MECHANICS TERMS

Some important terms and concepts in mechanics:

- **Energy** – symbol *E* - The ability to do work - unit is joule (J) or newton metre (Nm)
- **Torque** – symbol τ (Greek small letter tau) – turning force – unit is newton metre (Nm)
- **Power** – symbol *P* – The rate that energy is transferred over time – unit is joule per second ($J.s^{-1}$) or watt (W)
- **Efficiency** – symbol η – the ratio of *useful work or power output* to *useful work or power input*. Efficiency is dimensionless.

- See also *Book D: DC fundamentals EE3103 - Student workbook - Mechanics part one of two page 4*

SPEED AND VELOCITY

SPEED AND VELOCITY

Speed is the *rate of change of distance (d) with respect to time (t)*.

$$v = \frac{\Delta d}{\Delta t}$$

Speed is normally measured in *metres per second* ($m.s^{-1}$), but is also measured in *kilometres per hour* ($km.h^{-1}$ *or km/h, common in vehicles*).

Velocity is similar to speed, but it has *direction information* as well e.g. 10 km/h vs. 10 km/h north. 10 km/h is a speed, 10 km/h north is a velocity.

Speed and velocity are sometimes interchangeable!

SPEED AND VELOCITY

An object moving at 1 m.s^{-1} moves 1 metre in 1 second.

An object moving at 1 km/h moves 1 km in 1 hour.

1 m.s^{-1} = 3.6 km.h^{-1}

1 km.h^{-1} = 0.277778 m.s^{-1}

So…

10 km/h is about 2.78 m.s^{-1}

50 km/h is about 13.9 m.s^{-1}

100 km/h is about 27.8 m.s^{-1}

SPEED AND VELOCITY - EXAMPLES

The equation for speed is:

$$v = \frac{\Delta d}{\Delta t}$$

What is the speed of the following objects:

1. Moved 50 m in 5 s
2. Moved 1 m in 0.002 s
3. Moved 4 m in 3 s
4. How fast is a car going that is travelling at 80 $m.s^{-1}$?

SPEED AND VELOCITY - EXAMPLES

The equation for speed is:

$$v = \frac{\Delta d}{\Delta t}$$

What is the speed of the following objects?

1. 10 $m.s^{-1}$
2. 500 $m.s^{-1}$
3. 1.333 $m.s^{-1}$
4. 288 km/h

ACCELERATION

ACCELERATION

Acceleration is the *rate of change of speed or velocity (v) with respect to time (t).*

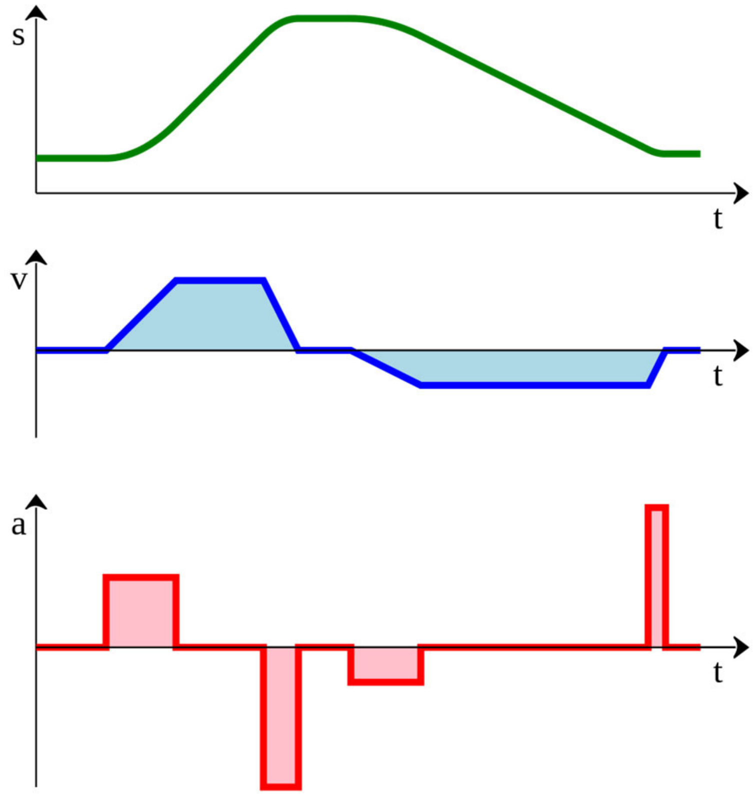
$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is normally measured in *metres per second squared* $(m.s^{-2})$.

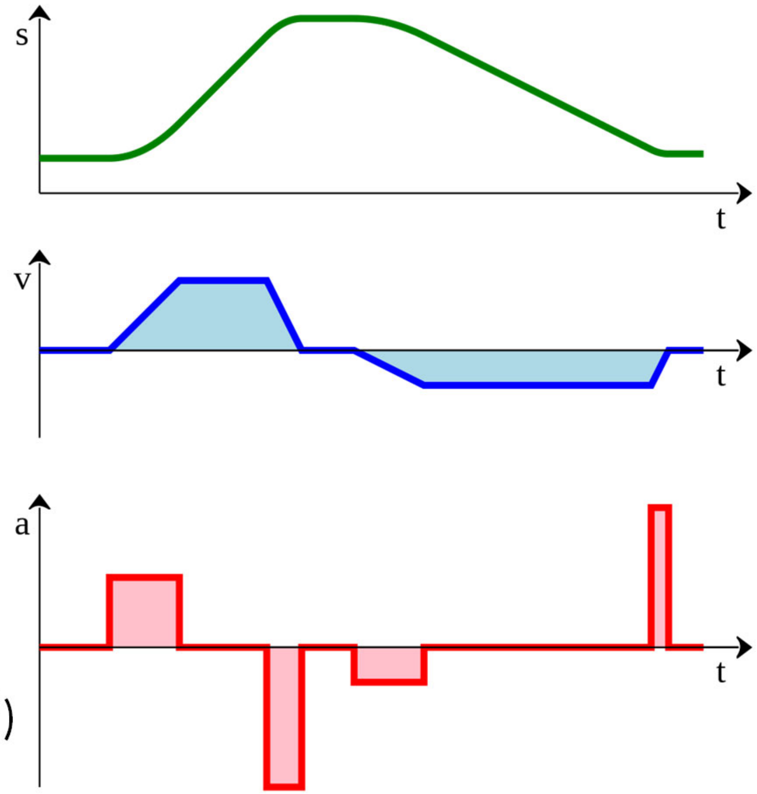
Accelerating a mass (m) requires a **force (F).**

$$F = ma$$

The graph on the right shows the distance (written *s*), speed and acceleration for the green path.



ACCELERATION - EXAMPLES



Acceleration is the *rate of change of change of speed or velocity (v) with respect to time (t).*

$$a = \frac{\Delta v}{\Delta t}$$

What is the acceleration of the following objects?

1. 0 to 9.81 $m.s^{-1}$ in 1 second?

2. 0 to 6 $m.s^{-1}$ in 2 seconds?

3. 0 to 100 km/h in 5 seconds? (give your answer in $m.s^{-2}$)

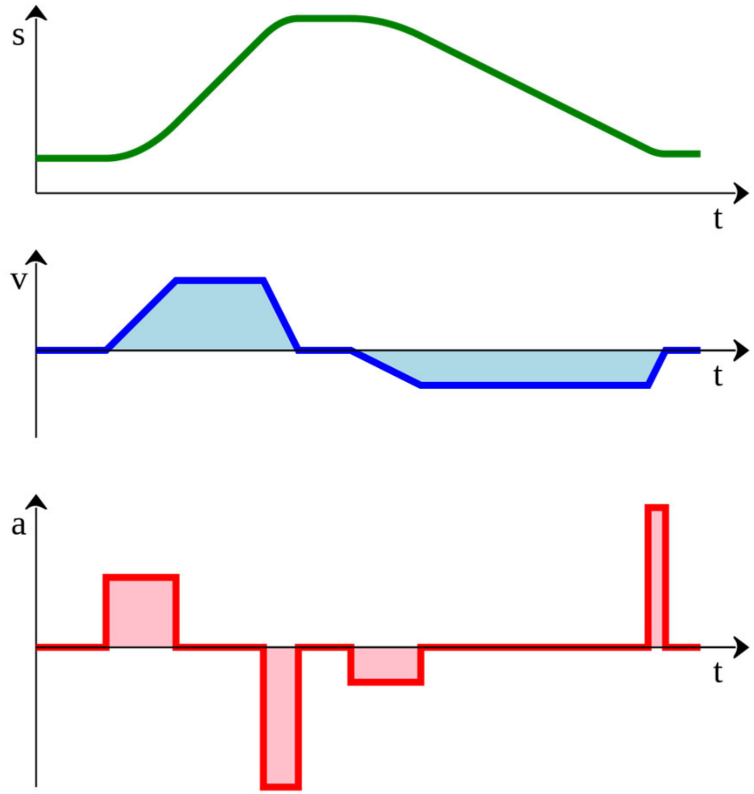
ACCELERATION - EXAMPLES

Acceleration is the *rate of change of change of speed or velocity (v) with respect to time (t).*

$$a = \frac{\Delta v}{\Delta t}$$

What is the acceleration of the following objects?

1. 9.81 $m.s^{-2}$
2. 3 $m.s^{-2}$
3. 5.556 $m.s^{-2}$



FORCE AND WEIGHT

FORCE AND WEIGHT

Force is an effect that tries to make an object change its speed. Force is the product if mass and acceleration.

$F = ma$

The unit of force is newton (N).

The most important force on earth is gravity. All objects have *weight* due to earth's gravity.

$F = mg$

Where *g* is the *acceleration due to gravity* (m.s^{-2}). On earth, g = 9.81 m.s^{-2}

FORCE AND WEIGHT - EXAMPLES

The equation for force and weight are:

$F = ma$, $F = mg$

Calculate the following:

1. The force on a 1000 kg mass accelerating at 5 m.s^{-2}?

2. The force on a 10 kg mass accelerating at 6 m.s^{-2}?

3. The weight of the kilogram mass at right.

4. The weight of the kilogram mass at right *on the moon* ($g = 1.62$ m.s^{-2}).

FORCE AND WEIGHT - EXAMPLES

The equation for force and weight are:

$F = ma, F = mg$

Calculate the following:

1. 5000 N
2. 60 N
3. 9.81 N
4. 1.62 N

WORK AND ENERGY

ENERGY

Energy is *the ability to do work*.

Energy has the symbol *E*, and is measured in joules (J).

If you have energy, you are able to do work.

Sometimes, energy and work are interchanged.

This is the case if doing work results in stored energy.

An example is doing work against gravity. The work that is done is available as stored energy that can itself do work. This is how hydro dams operate.

In a car, the petrol stores energy, but the expanding hot gas pushing against the piston does work.

WORK

Work is the amount of energy that is required to do a process through *physical movement against a force.*

Work *transforms energy from one form to another.*

Things that require work:

- Lifting masses against gravity.
- Speeding up or slowing down any object with mass.
- Overcoming air resistance.

WORK AND FORCE

Work (W) is the product of force (F) and distance moved (d).

$W = Fd$

For a force in newtons, and a distance in metres, the resulting work will be in joules (J), or newton metres (Nm).

Anytime you lift a mass, you do work. The formula for work against gravity is

$W = mgh$

Where W is the work done (J), *m* is the mass lifted, *g* is the acceleration due to gravity, and *h* is the height difference (m).

WORK AND FORCE - EXAMPLES

Work (W) is the product of force (F) and distance moved (d).

$W = Fd$, or $W = mgh$

What is the work done in the following situations (assume g = 9.81 m.s^{-2})?

1. A 75kg person climbs to the top of a 40 m tall building.
2. A dragster is accelerated over 40 m with a thrust force of 10000 N.
3. A weightlifter lifts a 200 kg weight bar 2.1 m.
4. A 75 kg person jumps 2 m high *on the moon* (g = 1.62 m.s^{-2}).

WORK AND FORCE - EXAMPLES

Work (W) is the product of force (F) and distance moved (d).

$W = Fd$, or $W = mgh$

What is the work done in the following situations (assume g = 9.81 m.s^{-2})?

1. 29430 J
2. 400000 J
3. 4120 J
4. 243 J

POWER

POWER

Power is the *rate of change of energy (E) or work (W) with respect to time (t).*

$P = \frac{\Delta E}{\Delta t}$, or $P = \frac{W}{t}$

Power is normally measured in *watts (W)*, or *joules per second ($J.s^{-1}$).*

Sometimes mechanical power is given in *horsepower* (hp).

1 hp = 746 W.

MECHANICAL POWER – FORCE-SPEED POWER

MECHANICAL POWER – FORCE-SPEED POWER

- Moving an object against at a force (*F*) at a certain speed (*v*) requires *power*.

$$P = Fv$$

- This formula is useful for calculating:
- the amount of power your car needs to overcome air resistance;
- The amount of power your car needs to *accelerate* (moving mass stores energy) as $P = mav$

MECHANICAL POWER – FORCE-SPEED POWER

- Moving an object against at a force (*F*) at a certain speed (*v*) requires *power*.

$P = Fv$, or $P = mav$

- Calculate the power required in the following situations:
- A vehicle travelling at 100 km/h needs to overcome 6000 N of drag. How much engine power is required to overcome the drag? Give your answers in kW and hp.
- A vehicle with a mass of 1000 kg is accelerating at 4 $m.s^{-2}$. How much engine power is required to accelerate the vehicle when it's travelling at 80 km/h? Give your answers in kW and hp.

MECHANICAL POWER – FORCE-SPEED POWER

- Moving an object against at a force (*F*) at a certain speed (*v*) requires *power*.

$P = Fv$, or $P = mav$

- Calculate the power required in the following situations:
- 166.7 kW or 223.4 hp
- 88.89 kW or 119.2 hp

MECHANICAL POWER – GRAVITY

MECHANICAL POWER - GRAVITY

- Lifting an object at a certain speed (v) against gravity requires *power*.

$$P = mgv$$

- Lifting an object against gravity in a certain amount of time requires *power.*

$$P = \frac{mgh}{t}$$

- These formulas are useful for calculating:
- The output power needed from a crane;
- Any situation where mass is being listed against gravity.

MECHANICAL POWER – GRAVITY EXAMPLES

- Lifting an object at a certain speed (*v*) against gravity requires *power*.

$P = mgv$, or $P = \frac{mgh}{t}$

- Calculate the power required in the following situations (assume g = 9.81 $m.s^{-2}$):
- A crane lifts a 500 kg load at 2 $m.s^{-1}$. How much power is required to lift the mass at this speed? Give your answer in kW and hp.
- A 70 kg person runs up stairs. The vertical component of their velocity is 1.5 $m.s^{-1}$. How much power do they need to output to do this? Give your answer in W and hp.
- A crane lifts a 2500 kg load 40 m in 30 s. How much power is required to lift a mass this distance in this amount of time? Give your answer in kW and hp.

MECHANICAL POWER – GRAVITY EXAMPLES

- Lifting an object at a certain speed (*v*) against gravity requires *power*.

$$P = mgv, \text{ or } P = \frac{mgh}{t}$$

- Calculate the power required in the following situations (assume g = 9.81 m.s^{-2}):
- 9810 W or 13.15 hp.
- 1030 W or 1.381 hp.
- 32.7 kW or 43.83 hp.

MECHANICAL POWER – GRAVITY

MECHANICAL POWER – MASS TRANSFER

- Transferring mass against gravity requires *power*.

$$P = \dot{m}gh$$

- The dot over the m means “rate with respect to time”. $\dot{m}$ has the unit *kilogram per second* ($kg.s^{-1}$).
- This formula is useful for calculating the amount of power required from a pump.
- Sometimes, a volume rate is used, such as litres per minute. To get $\dot{m}$, you need to know the *density* of the fluid. For water, the density is 1000 $kg.m^{-3}$ (1000 kg per cubic metre), or 1 $kg.L^{-1}$ (1 kilogram per litre).
- You may also have to convert from “per minute” rates to “per second” rates. 1 min = 60 s.

MECHANICAL POWER – MASS TRANSFER

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MECHANICAL POWER – MASS TRANSFER EXAMPLES

- Lifting an object at a certain speed (*v*) against gravity requires *power*.

$$P = \dot{m}gh$$

- Calculate the power required in the following situations (assume g = 9.81 $m.s^{-2}$):
- A pump has a capacity of 60 $L.min^{-1}$. How much power is required to pump water at this rate over a 20 m rise? Give your answer in kW and hp.
- A washing machine requires its 100 L capacity bowl to be emptied in 60 s. The required lift distance is 0.8 m. How much power is required? Give your answer in W and hp.
- During construction of a basement, water infiltrates the basement at the rate of 5000 $L.min^{-1}$. The water is to be pumped from 15 m underground. How much pump power is required? Give your answer in kW and hp.

MECHANICAL POWER – MASS TRANSFER EXAMPLES

- Lifting an object at a certain speed (*v*) against gravity requires *power*.

$$P = \dot{m}gh$$

- Calculate the power required in the following situations (assume g = 9.81 $m.s^{-2}$):
- 196.2 W or 0.263 hp
- 13.08 W or 0.01753 hp
- 12.26 kW or 16.44 hp

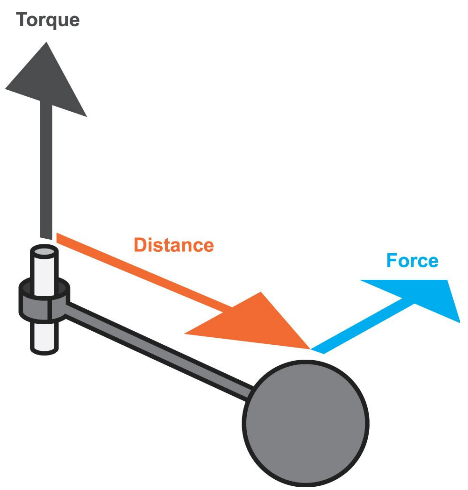
TORQUE

TORQUE

- Torque (τ or "tau") is the effect of exerting a *turning force.*

$$\tau = Fr$$

- r is the *radius* (m), and F is the force (N)
- The force *F* is *at right angles to the radius.*
- Torque is the equivalent of force for rotating machinery.
- The output is in newton metres (Nm)
- Torque may also be called *moment* or *couple*

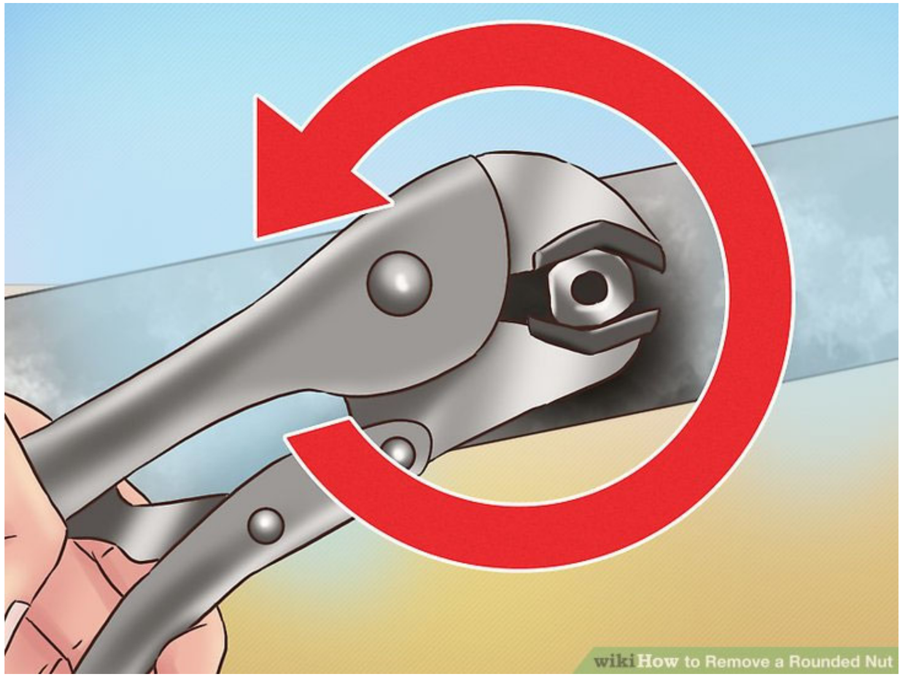


TORQUE - EXAMPLES

- Torque (τ or "tau") is the effect of exerting a *turning force.*

$$\tau = Fr$$

- Calculate the torque produced in the following situations (assume $g = 9.81$ m.s^{-2}):
- A force of 250 N exerted over a radius of 0.5 m.
- A force of 5000 N exerted over 0.025 m.
- A force of 1 N exerted over 1 m.
- A 5kg mass hanging freely off a pulley wheel with a radius of 0.25 m.



TORQUE - EXAMPLES

- Torque (τ or "tau") is the effect of exerting a *turning force.*

$$\tau = Fr$$

- Calculate the torque produced in the following situations (assume g = 9.81 $m.s^{-2}$):
- 125 Nm
- 125 Nm
- 1 Nm
- 12.26 Nm

TORQUE AND WORK

TORQUE AND WORK

- Torque on its own *does not do any work*, because the force is at right angles to the distance.
- To get the work from torque, you have to know the *total distance around the circumference of the circle the torque force acted on*. Luckily, we can get this from radius and angular displacement alone. The work done by a torque, for a angle (θ) in *degrees* is shown below.

$W = \tau \cdot \frac{\pi\theta}{180}$, or $W = Fr \cdot \frac{\pi\theta}{180}$

- Calculate the work done by a torque in the following situations (1 rev = 360°). Give your answer in joules (J) or kilojoules (kJ) as appropriate:
- 500 Nm over 100 revs
- 125 Nm over 90°
- 1 Nm over 57.3°
- Suppose a car is parked on a hill with the handbrake engaged. The handbrake exerts a torque that prevents the car from rolling down the hill. Is the handbrake torque doing any work?

TORQUE AND WORK

- Torque on its own *does not do any work*, because the force is at right angles to the distance.
- To get the work from torque, you have to know the angular displacement the torque has been exerted over. The equation, for a angle (θ) in *degrees* is shown below.

$W = \tau \cdot \frac{\pi\theta}{180}$, or $W = Fr \cdot \frac{\pi\theta}{180}$

- Calculate the work done by a torque in the following situations (1 rev = 360°). Give your answer in joules (J) or kilojoules (kJ) as appropriate:
- 314.2 kJ
- 196.3 J
- 1 J
- No, if the car is not moving, there is no change in angle in the wheels.

TORQUE AND POWER

TORQUE AND POWER

- To get the power from torque, you have to know the *rate of angular displacement*. A common measure is revolutions per minute (rpm). The equation, for rate *N* (rpm) is shown below.

$$P = \frac{2\pi N\tau}{60}$$

- This formula is useful for calculating motor output power, for example.
- Calculate the power output of the following rotating machines. Give your answers in watts (or kilowatts if appropriate) and horsepower:
- 20 Nm at 1000 rpm
- 125 Nm at 5000 rpm
- 4.5 Nm at 2980 rpm

TORQUE AND POWER

- To get the power from torque, you have to know the *rate of angular displacement*. A common measure is revolutions per minute (rpm). The equation, for rate *N* (rpm) is shown below.

$$P = \frac{2\pi N\tau}{60}$$

- This formula is useful for calculating motor output power, for example.
- Calculate the power output of the following rotating machines. Give your answers in watts (or kilowatts if appropriate) and horsepower:
- 2094 W or 2.094 kW or 2.808 hp
- 65.44 kW or 87.73 hp
- 1404 W or 1.404 kW or 1.882 hp