SERIES AND PARALLEL CIRCUITS

A Companion to Book “A”

INTRODUCTION

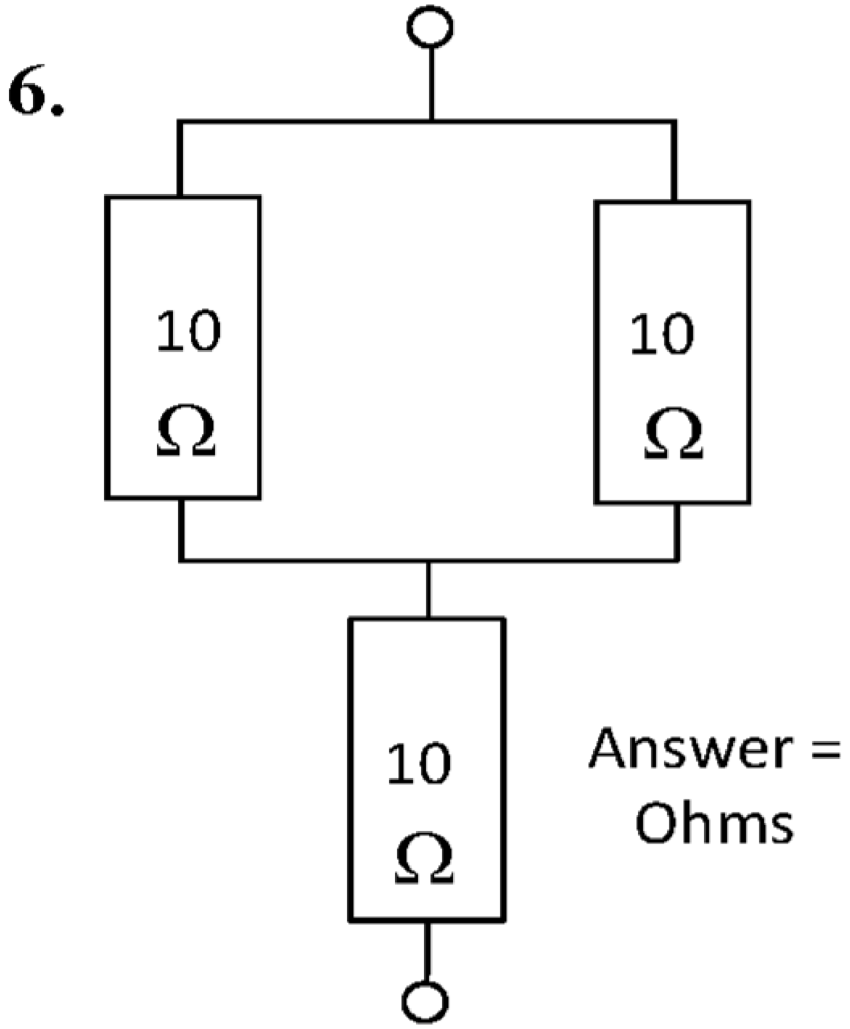
Electrical circuits can be arranged in various ways.

This set of slides discusses series and parallel resistive circuits.

Reference: Book A – Kirchhoff's Law Problems

Shown at right:

Worksheet 10B Question 6.



SERIES CIRCUITS

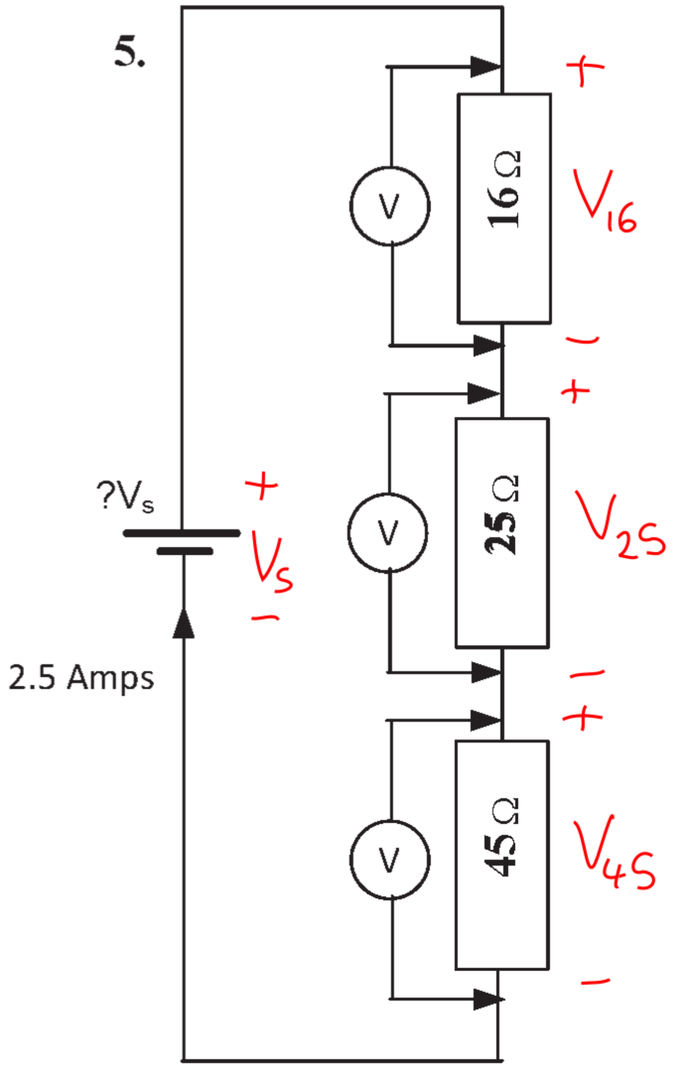
SERIES CIRCUITS

Series circuits have the *same current* flowing through multiple components.

The *voltage drop across all the components is the same as the sum of the voltage drops across each individual component.*

This is just Kirchhoff's Voltage Law!

Shown at right: Worksheet 2A Question 5.



V_S =

VD_{25} =

VD_{16}=

VD_{45} =

SERIES CIRCUITS

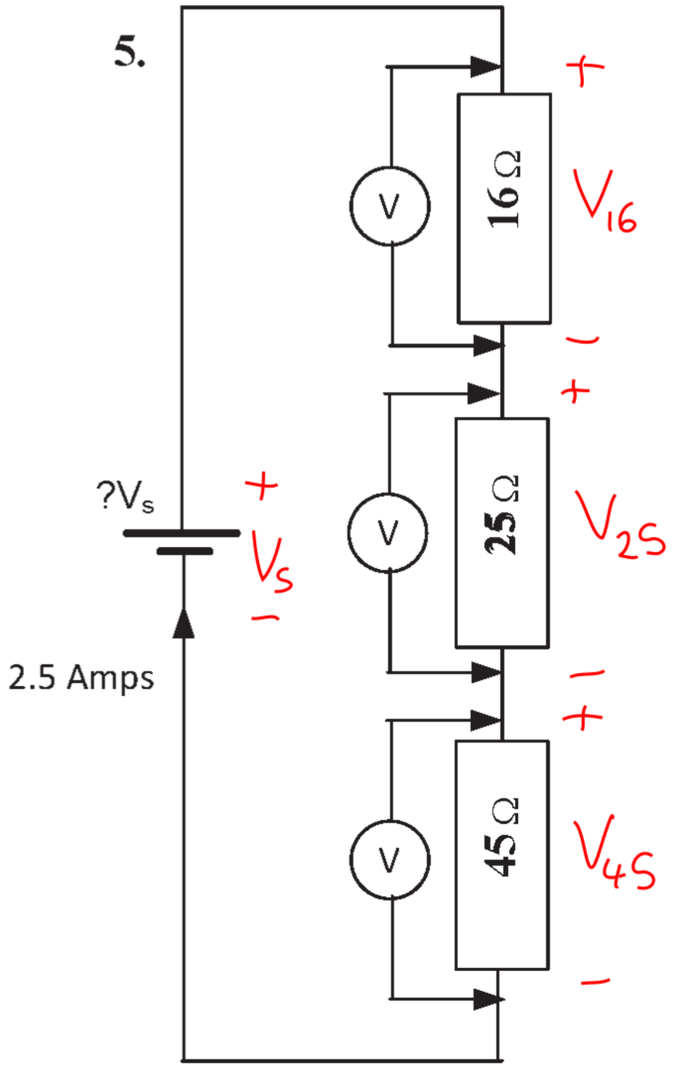
Kirchhoff's Voltage Law is:

$$V_T = V_1 + V_2 + V_3 + \cdots + V_n$$

where V_T is the *total voltage drop*, and V_1 through V_n are the voltage drops in the series network.

The voltages may have different subscripts.

$$V_s = V_{16} + V_{25} + V_{45}$$



V_S =

VD_{25} =

VD_{16}=

VD_{45} =

SERIES CIRCUITS - RESISTORS

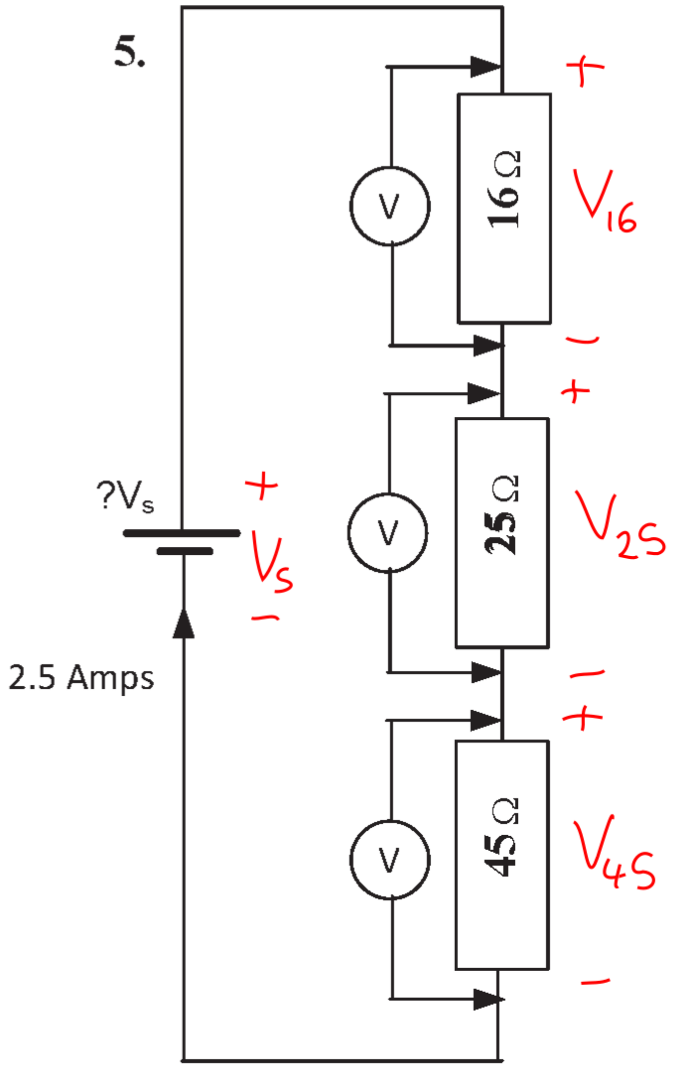
For resistors, the series formula is:

$$R_T = R_1 + R_2 + R_3 + \cdots + R_n$$

where R_T is the *total resistance*, and R_1 through R_n are the resistors in the series network.

This formula comes from Ohm's Law and Kirchhoff's Law.

What is the total series resistance of the circuit at right?



V_S =

VD_{25} =

VD_{16}=

VD_{45} =

SERIES CIRCUITS - RESISTORS

For resistors, the series formula is:

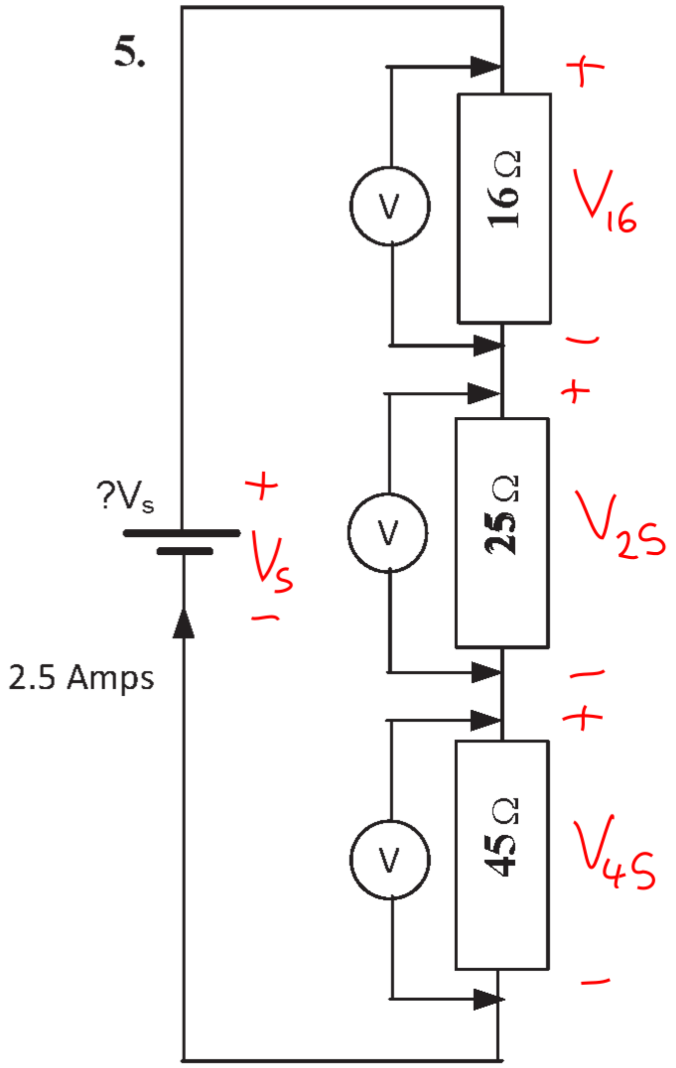
$$R_T = R_1 + R_2 + R_3 + \cdots + R_n$$

where R_T is the *total resistance*, and R_1 through R_n are the resistors in the series network.

This formula comes from Ohm's Law and Kirchhoff's Voltage Law.

$$R_T = 45 + 25 + 16 = 86\ \Omega$$

5.



V_S =

VD_{25} =

VD_{16}=

VD_{45} =

SERIES CIRCUITS - RESISTORS

For resistors, the series formula derived from KVL using Ohm's Law:

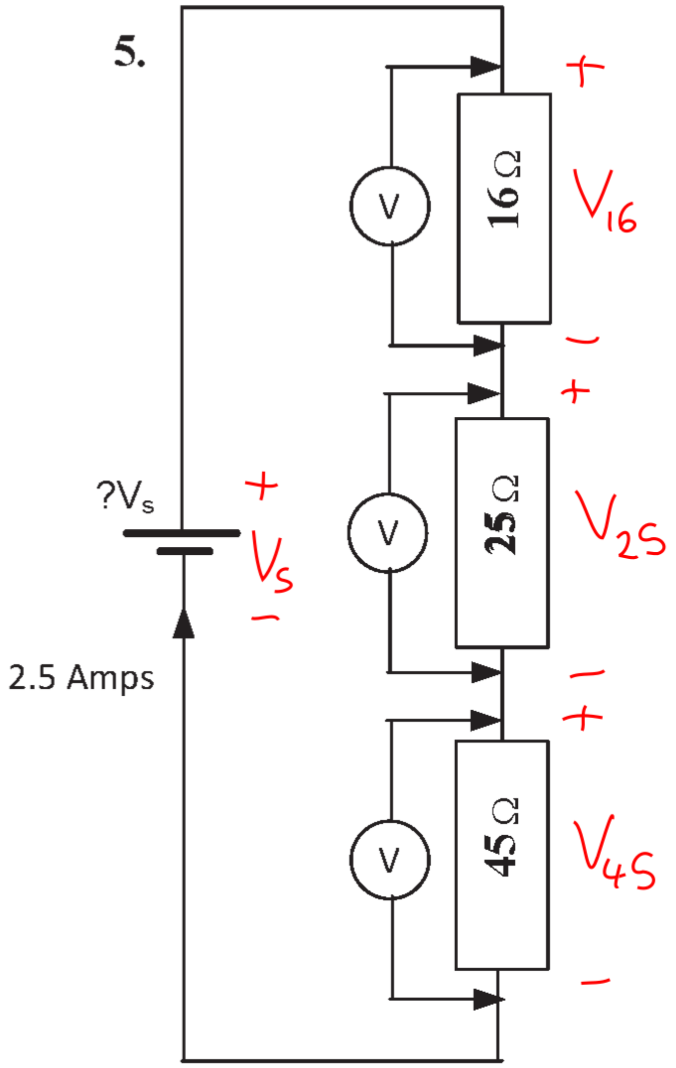
$$IR_T = IR_1 + IR_2 + IR_3 + \cdots + IR_n$$

where I is the current flowing through the series network.

The I terms cancel out, leaving the series resistance formula.

The formula can be used to calculate total voltage drop:

$$V_T = IR_1 + IR_2 + IR_3 + \cdots + IR_n$$



V_S =

VD_{25} =

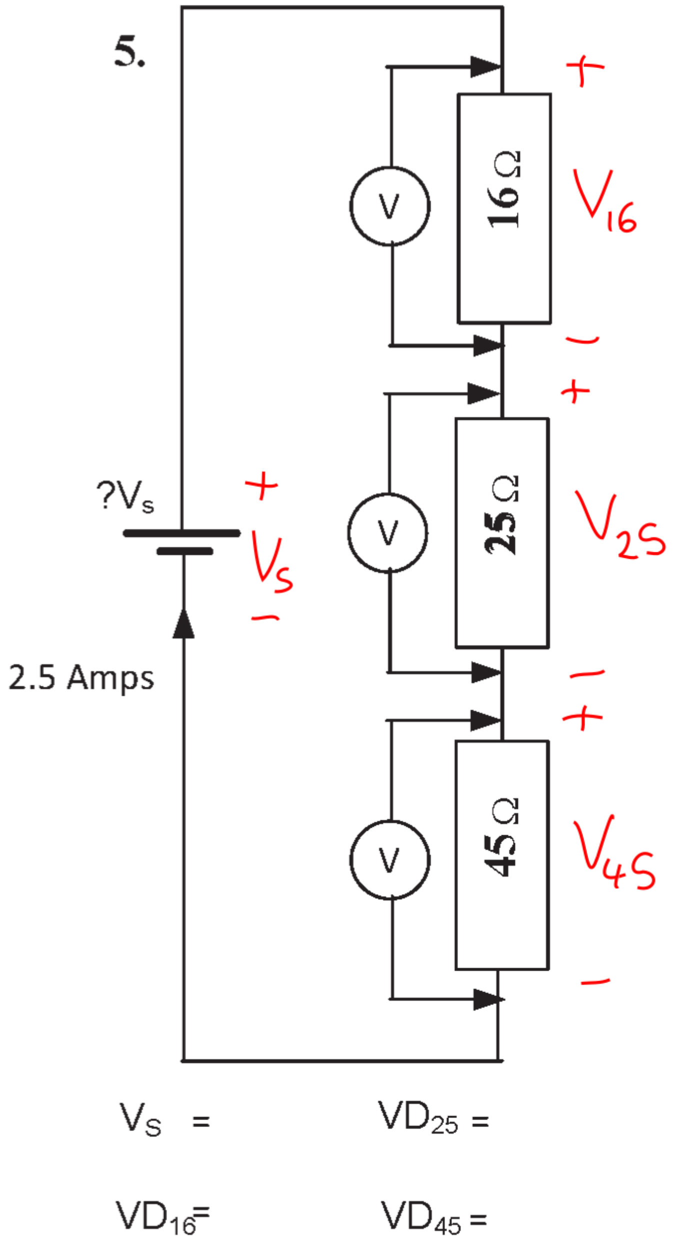
VD_{16}=

VD_{45} =

SERIES CIRCUITS - RESISTORS

What is the total voltage drop across the resistors (i.e. V_S) in the circuit at right?

Note the current flow is 2.5 A.



SERIES CIRCUITS - RESISTORS

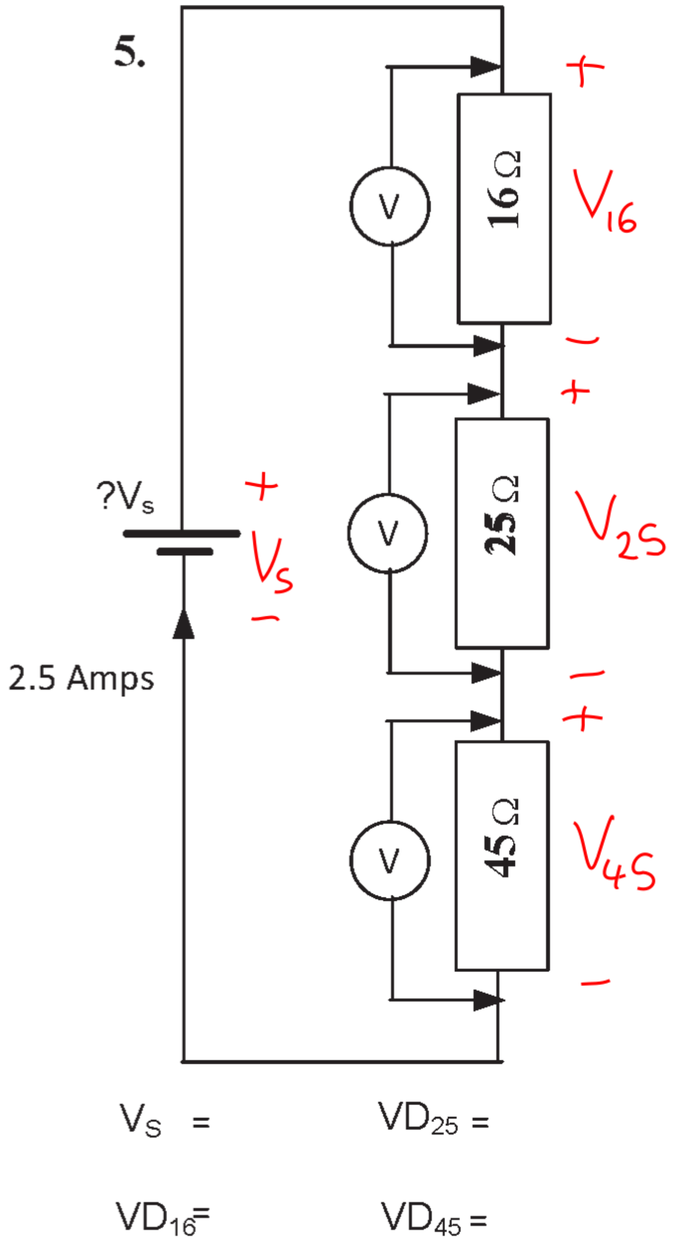
What is the total voltage drop across the resistors (i.e. V_S) in the circuit at right?

Note the current flow is 2.5 A.

$$V_S = 2.5 \cdot 45 + 2.5 \cdot 25 + 2.5 \cdot 16$$

$$V_S = 112.5 + 62.5 + 40$$

$$V_S = 215 \text{ V}$$



RELEVANT WORKSHEETS

The following worksheets are the most relevant to series circuits:

- Worksheets 2A, 2B, 3A, 10A

VOLTAGE DIVISION

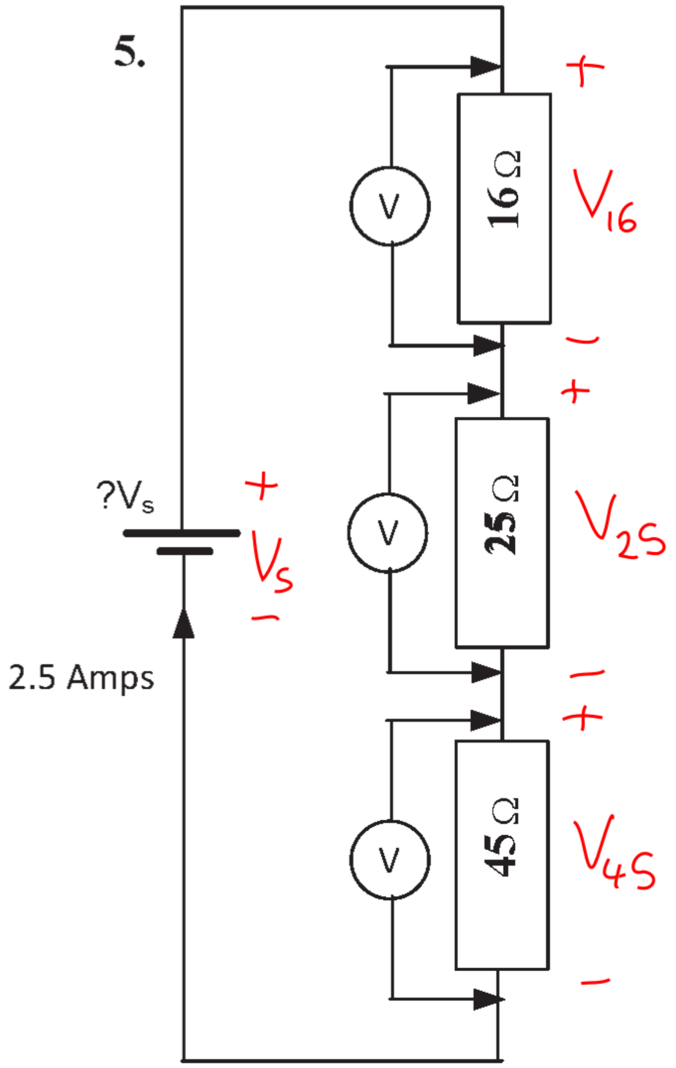
VOLTAGE DIVIDER

A series circuit forms a *voltage divider* – the total voltage "divides" according to the *total resistance*, and the *resistor of interest*.

The voltage divider equation is:

$$V_m = V_T \cdot \frac{R_m}{R_T}$$

where V_m is the voltage drop of interest, V_T is the *total series voltage drop*, R_m is the *resistor of interest*, and R_T is the *total series resistance*.



V_S =

VD_{25} =

VD_{16}=

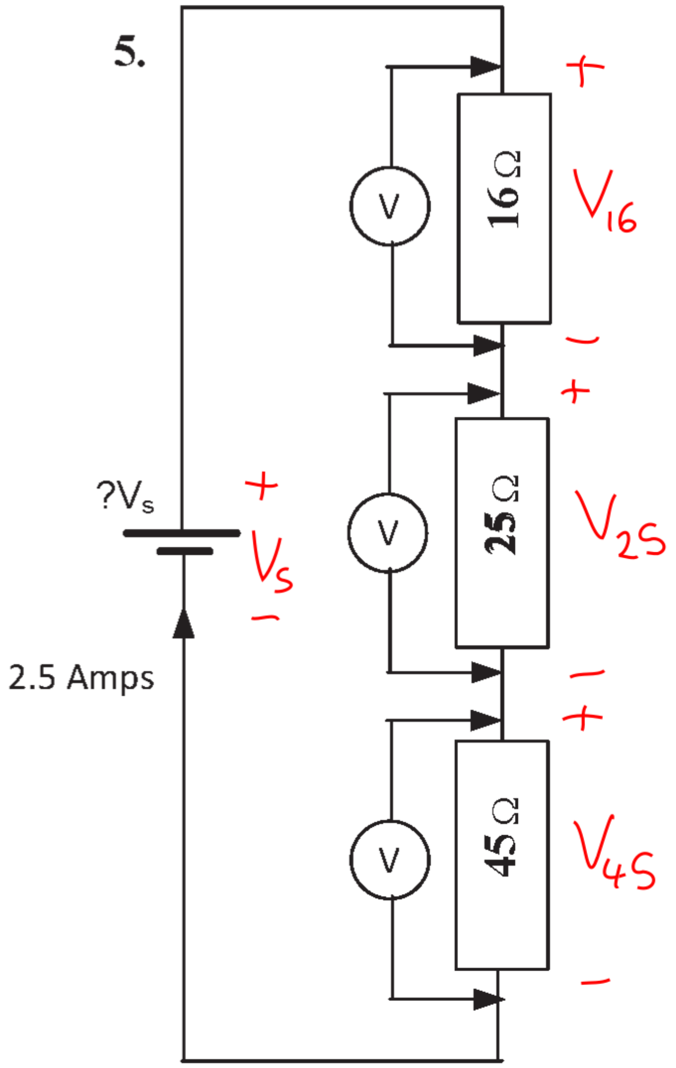
VD_{45} =

VOLTAGE DIVIDER

The voltage divider equation is:

$$V_m = V_T \cdot \frac{R_m}{R_T}$$

Use the voltage divider equation to calculate the voltage drops across the resistors as shown, given the supply voltage is 215 V.



V_S =

VD_{25} =

VD_{16} =

VD_{45} =

VOLTAGE DIVIDER

The voltage divider equation is:

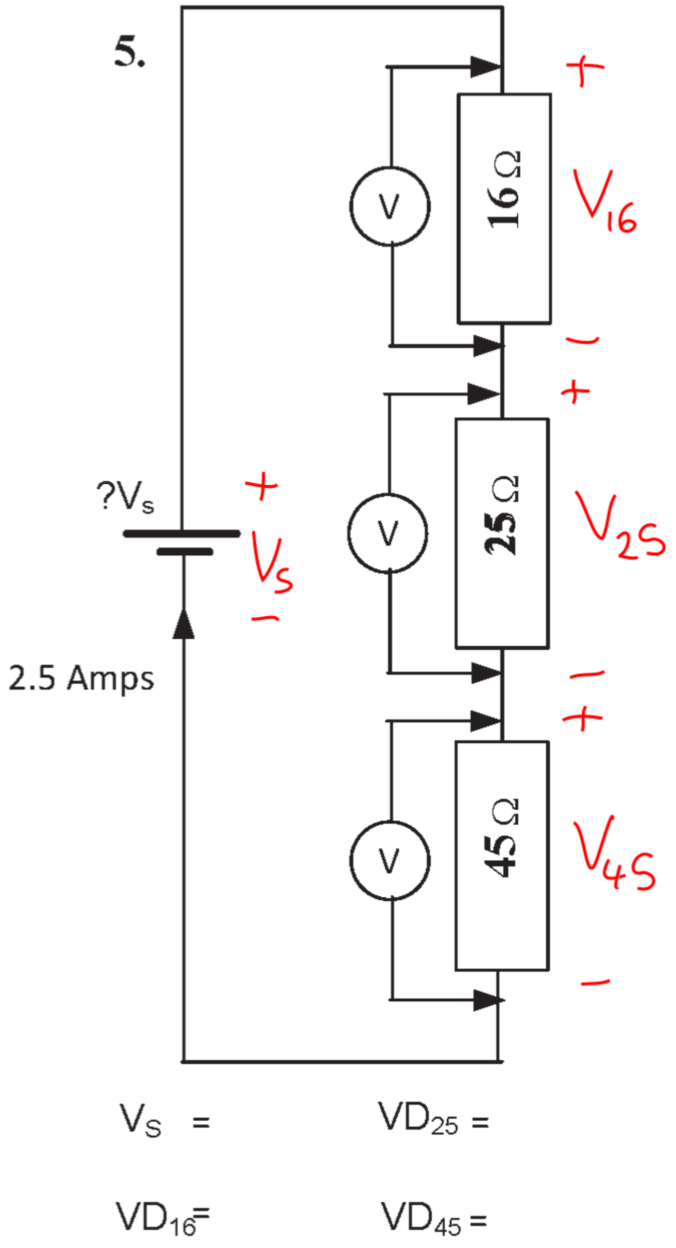
$$V_m = V_T \cdot \frac{R_m}{R_T}$$

Use the voltage divider equation to calculate the voltage drops across the resistors as shown, given the supply voltage is 215 V.

$$V_{16} = 215 \cdot \frac{16}{86} = 40 \text{ V}$$

$$V_{25} = 215 \cdot \frac{25}{86} = 62.5 \text{ V}$$

$$V_{45} = 215 \cdot \frac{45}{86} = 112.5 \text{ V}$$



PARALLEL CIRCUITS

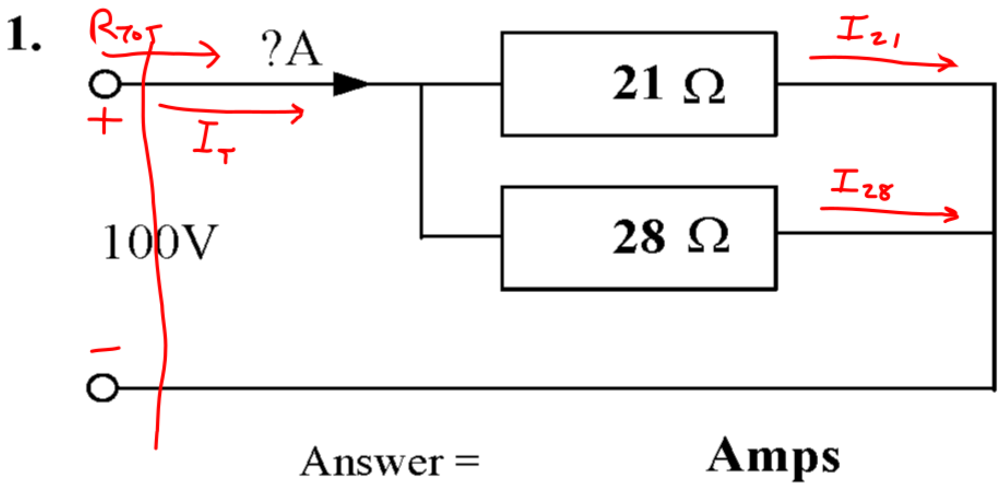
PARALLEL CIRCUITS

Parallel circuits have the *same voltage drop* across through components.

The *current flow through all the components is the same as the sum of the current flows through each individual component.*

This is just Kirchhoff's Current Law!

Shown at right: Worksheet 5A Question 1.



PARALLEL CIRCUITS

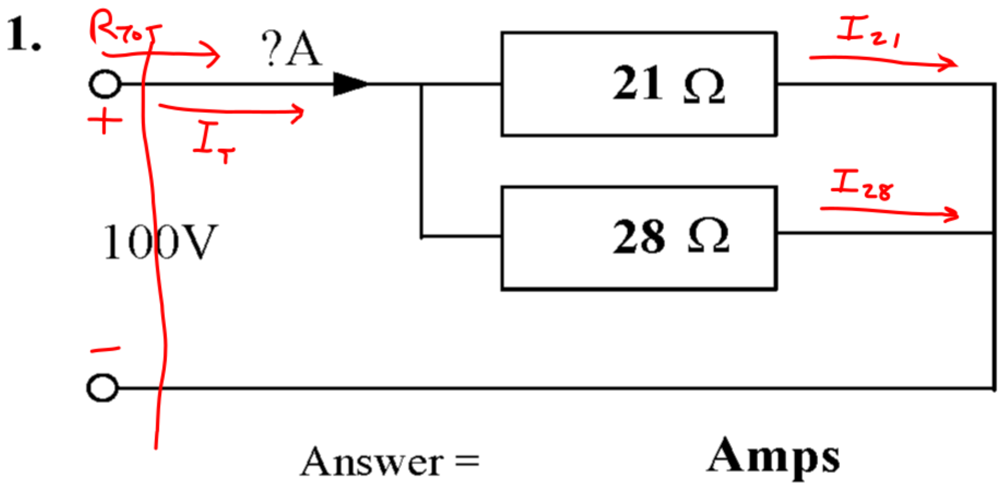
Kirchhoff's Current Law is:

$$I_T = I_1 + I_2 + I_3 + \cdots + I_n$$

where I_T is the *total current flow*, and I_1 through I_n are the current flows in the parallel network.

The currents may have different subscripts.

$$I_T = I_{21} + I_{28}$$



PARALLEL CIRCUITS - RESISTORS

For resistors, the parallel formula is:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

where R_T is the *total resistance*, and R_1 through R_n are the resistors in the parallel network.

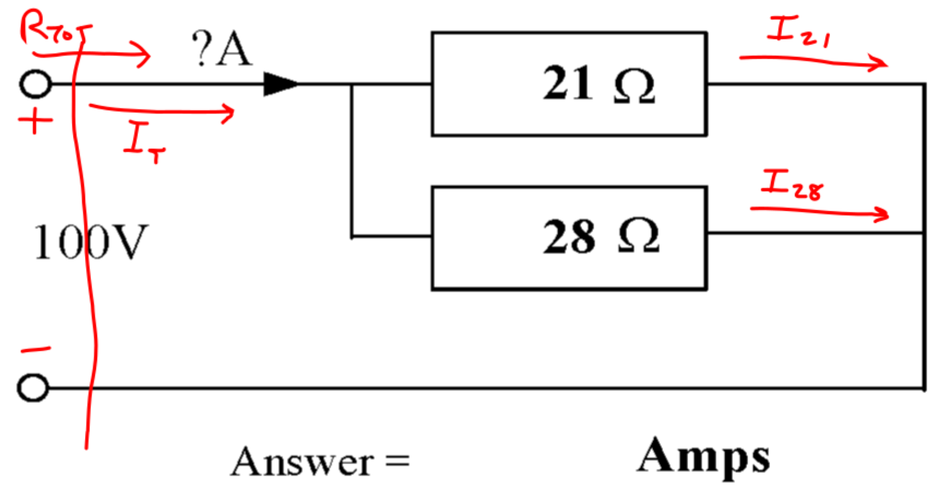
This formula may be written as:

$$R_T = R_1 \| R_2 \| R_3 \| \cdots \| R_n$$

This formula comes from Ohm's Law and Kirchhoff's Current Law.

What is the total parallel resistance of the circuit at right?

1.



PARALLEL CIRCUITS - RESISTORS

For resistors, the parallel formula is:

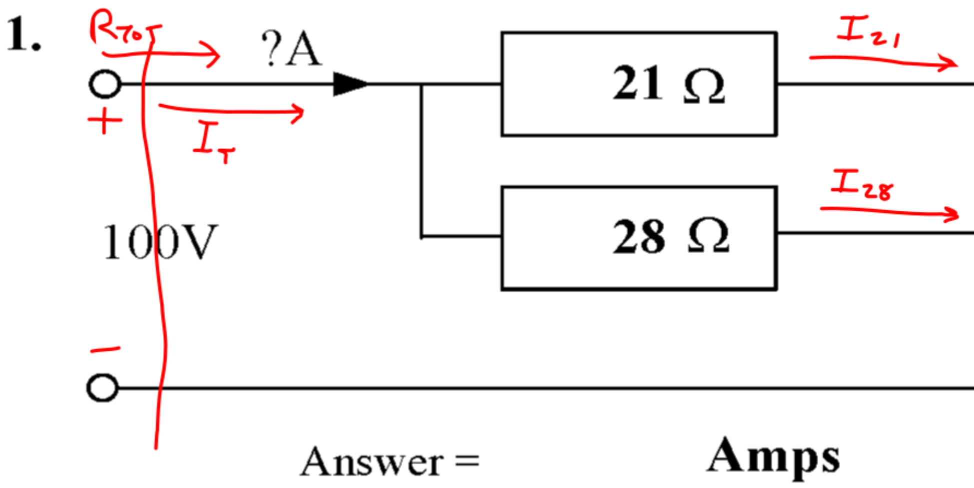
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

where R_T is the *total resistance*, and R_1 through R_n are the resistors in the parallel network.

This formula comes from Ohm's Law and Kirchhoff's Current Law.

$$\frac{1}{R_{TOT}} = \frac{1}{21} + \frac{1}{28} = \frac{1}{12}$$

$$R_{TOT} = 12\ \Omega$$



PARALLEL CIRCUITS - RESISTORS

For resistors, the series formula derived from KCL using Ohm's Law:

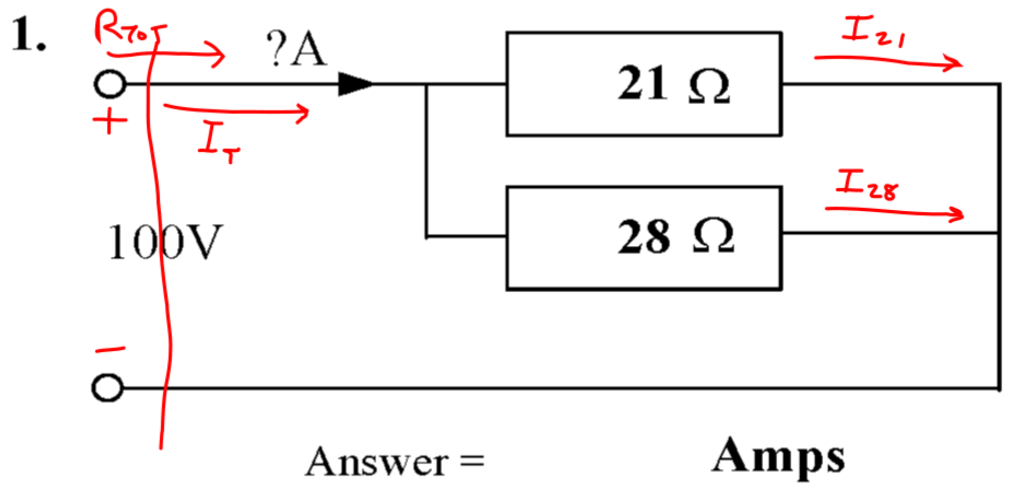
$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \frac{V}{R_n}$$

where V is the voltage across the parallel network.

The V terms cancel out, leaving the parallel resistance formula.

The formula can be used to calculate total current flow:

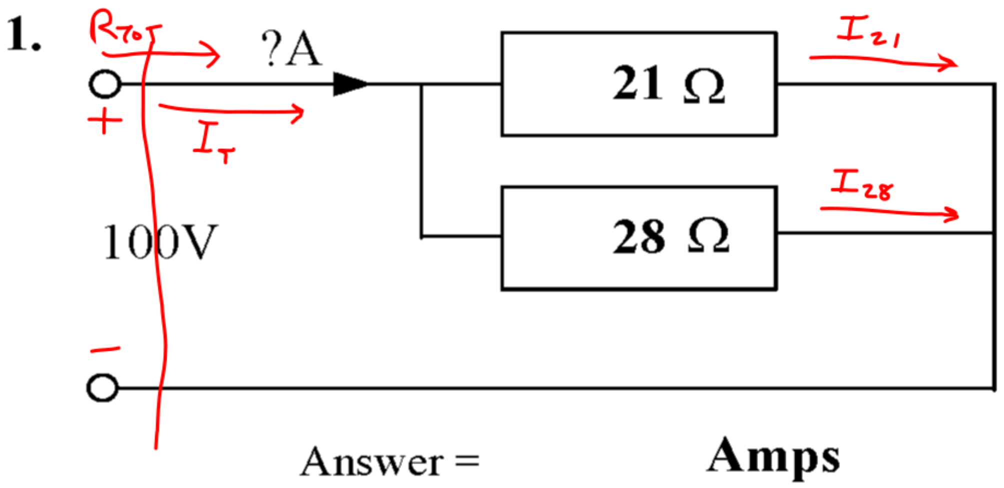
$$I_T = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots + \frac{V}{R_n}$$



PARALLEL CIRCUITS - RESISTORS

What are current flows in the circuit at right?

Note the supply voltage is 100 V.



PARALLEL CIRCUITS - RESISTORS

What are current flows in the circuit at right?

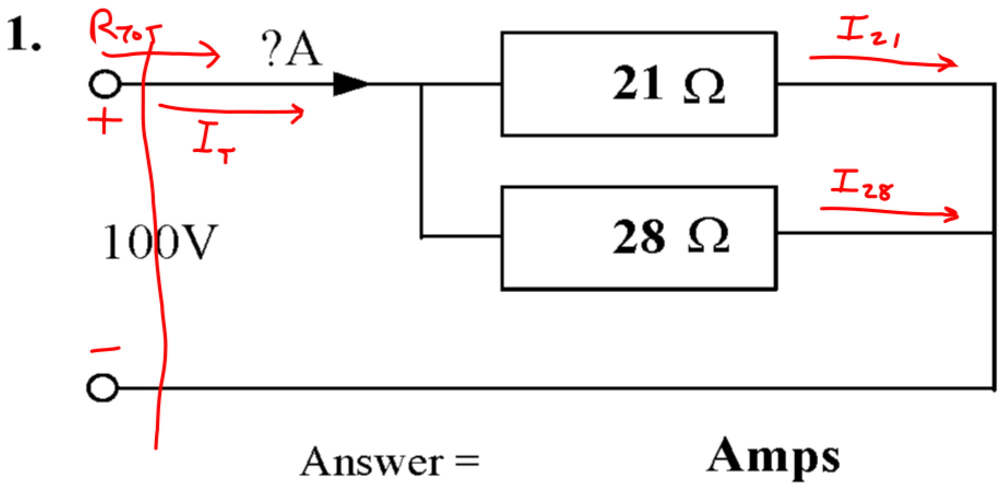
Note the supply voltage is 100 V.

$I_{21} = \frac{100}{21} = 4.7619$ A (or 4.762 A)

$I_{28} = \frac{100}{28} = 3.57143$ A (or 3.571 A)

$I_T = 4.7619 + 3.57143 = 8.333$ A

Note that this equals $\frac{100}{21\|28}$



RELEVANT WORKSHEETS

The following worksheets are relevant to parallel circuits:

- Worksheets 5A, 6A, 10B, 10C

CURRENT DIVISION

CURRENT DIVIDER

A parallel circuit forms a *current divider* – the total current "divides" according to the *total resistance*, and the *resistor of interest*.

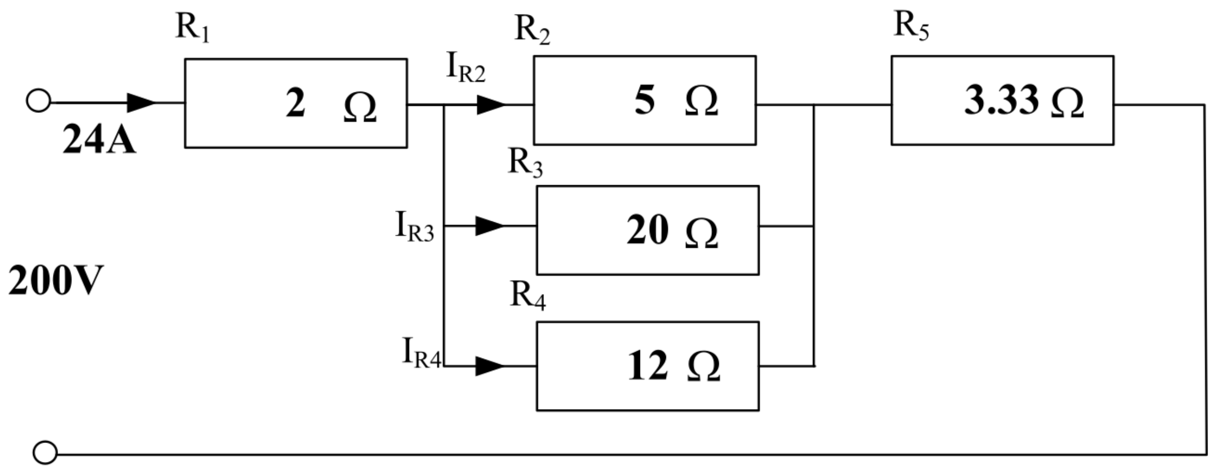
The current divider equation is:

$$I_m = I_T \cdot \frac{R_T}{R_m}$$

where I_m is the current flow of interest, I_T is the *total parallel current*, R_m is the *resistor of interest*, and R_T is the *total parallel resistance*.

Shown at right: Worksheet 15A Q4

4.



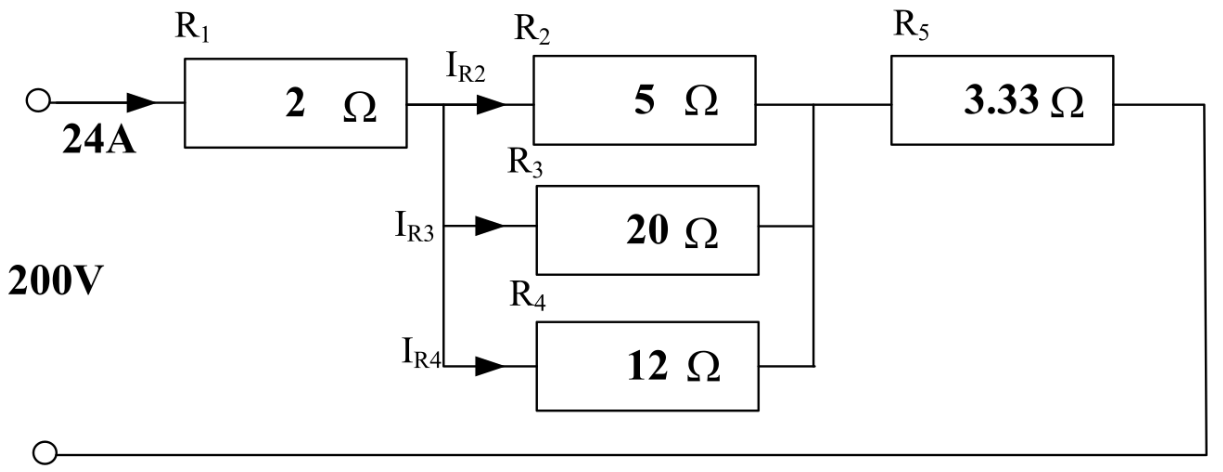
CURRENT DIVIDER

The current divider equation is:

$$I_m = I_T \cdot \frac{R_T}{R_m}$$

Use the current divider equation to calculate the currents through R_1, R_2, and R_3, given the total current flow of 24 A.

4.



CURRENT DIVIDER

The current divider equation is:

$$I_m = I_T \cdot \frac{R_T}{R_m}$$

Use the current divider equation to calculate the currents through R_1, R_2, and R_3, given the total current flow of 24 A.

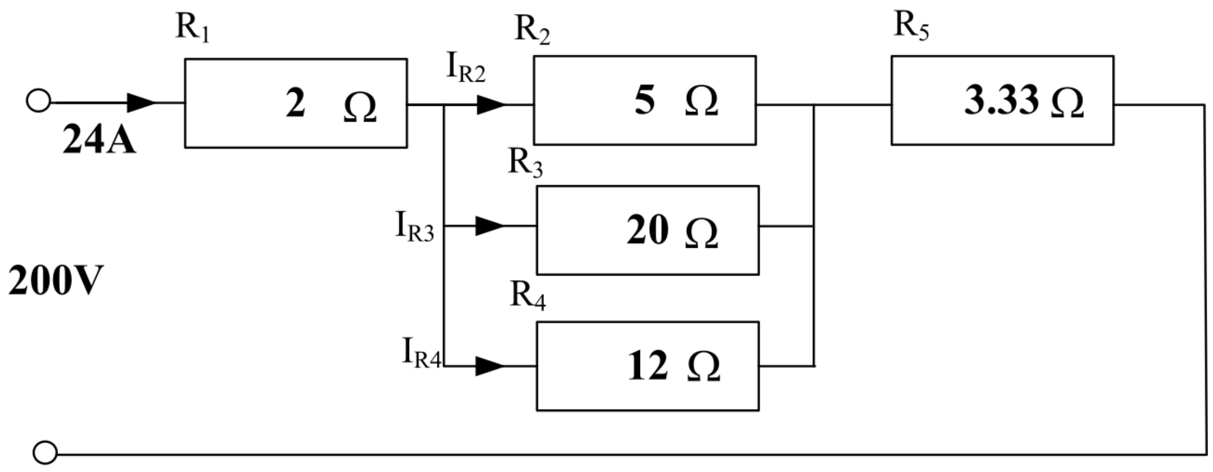
$$R_T = 5\|20\|12 = 3\ \Omega$$

$$I_{R2} = 24 \cdot \frac{3}{5} = 14.4\text{ A}$$

$$I_{R3} = 24 \cdot \frac{3}{20} = 3.6\text{ A}$$

$$I_{R4} = 24 \cdot \frac{3}{12} = 6\text{ A}$$

4.



RELEVANT WORKSHEETS

The following worksheets are relevant to current division:

- Worksheets 15A, 15B, 16A, 16C

SERIES-PARALLEL CIRCUITS

INTRODUCTION

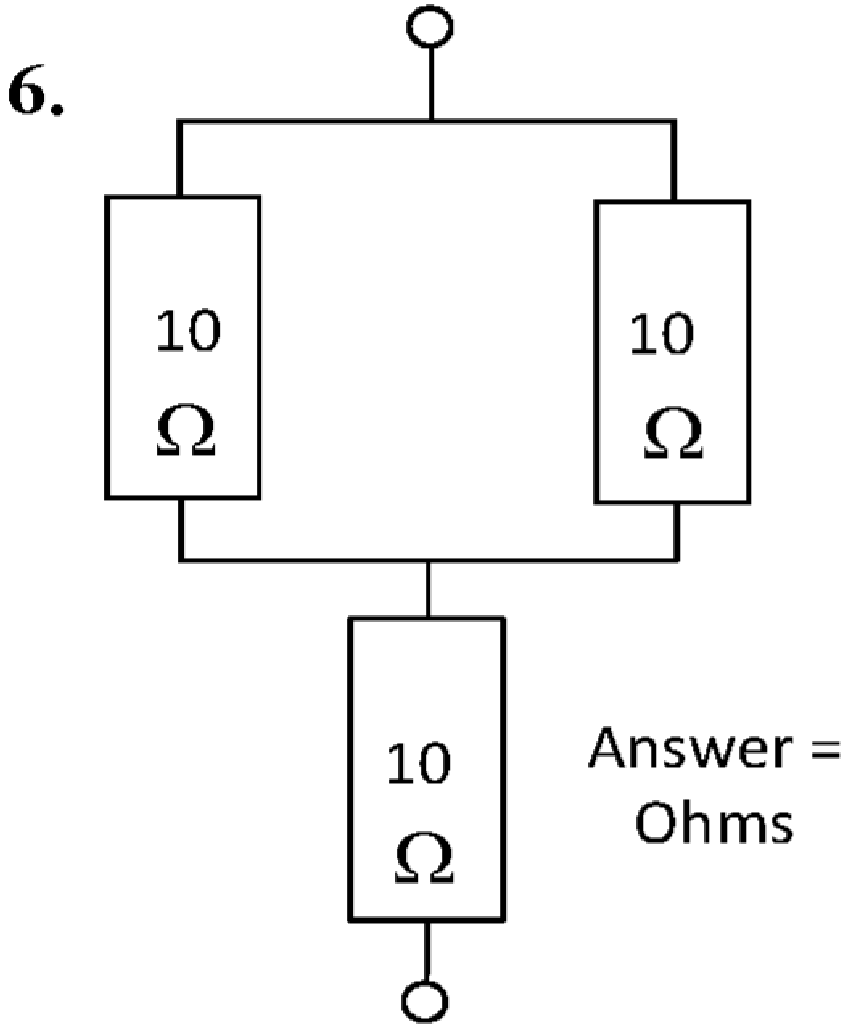
Electrical circuits can be arranged in various ways.

This set of slides discusses series and parallel resistive circuits.

Reference: Book A – Kirchhoff's Law Problems

Shown at right:

Worksheet 10B Question 6.



SERIES-PARALLEL CIRCUITS

Series-parallel circuits are generally too complex to have a single formula.

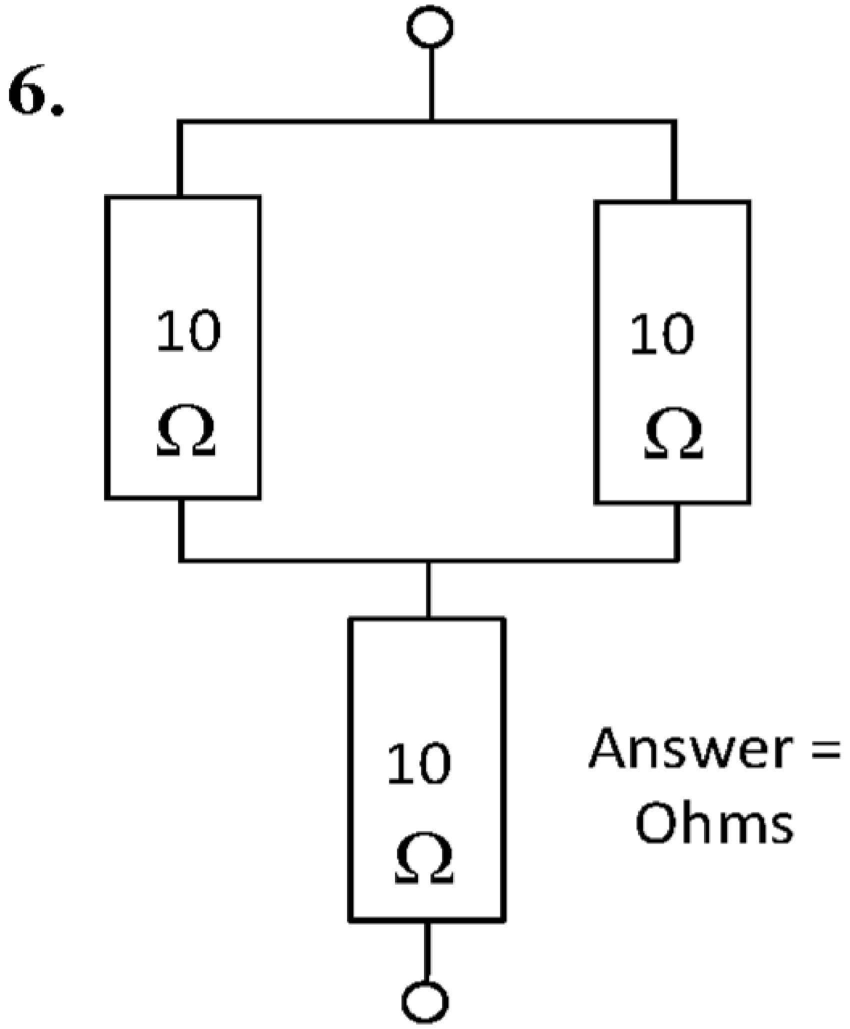
Instead, they are *broken down* into simple series and parallel circuits.

They are then "merged" into a single resistor using series and parallel resistance formulas.

Relevant Worksheets: 10A, 10B, 10C, 10D, 15A, 15B, 16A, 16C, 16D, 16E

SERIES-PARALLEL CIRCUITS

The objective is to find the total resistance of the circuit at right.



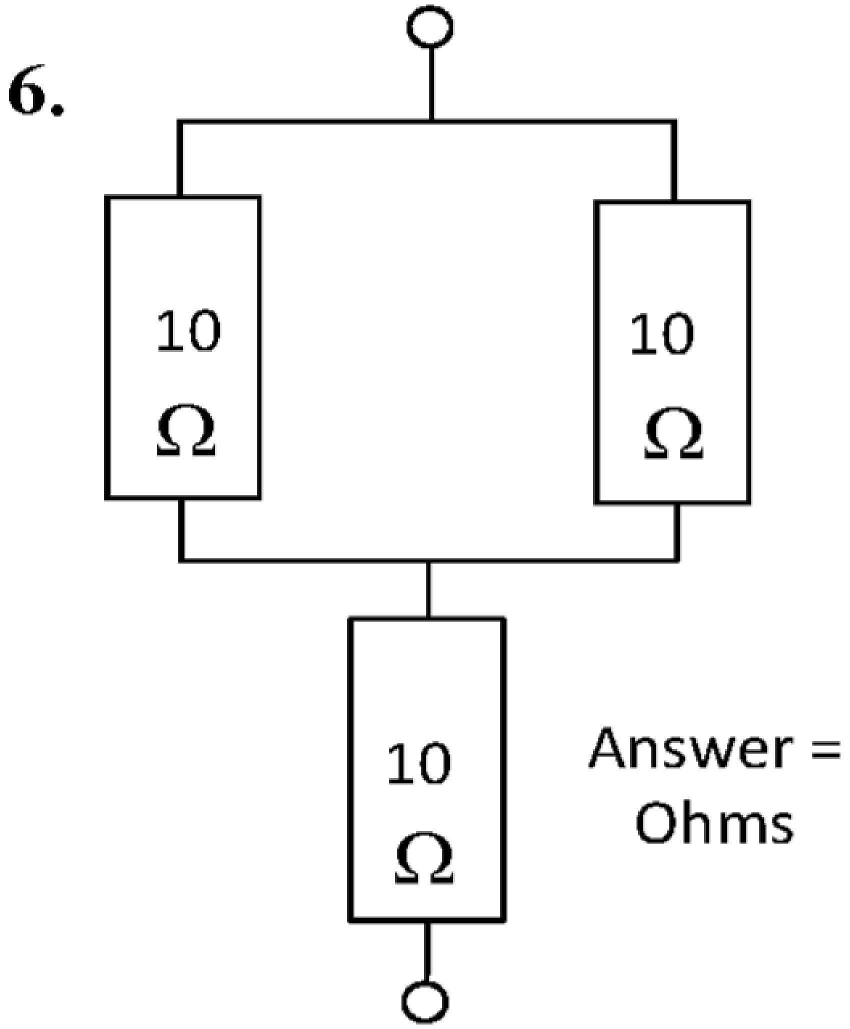
SERIES-PARALLEL CIRCUITS

The circuit consists of two "subcircuits":

- Two 10 Ω resistors in parallel; and
- A 10 Ω resistor in series.

Do not assume anything about how the current "splits" or merges without a reason!

Do not assume that resistors in different networks "know about" each other. They do not.



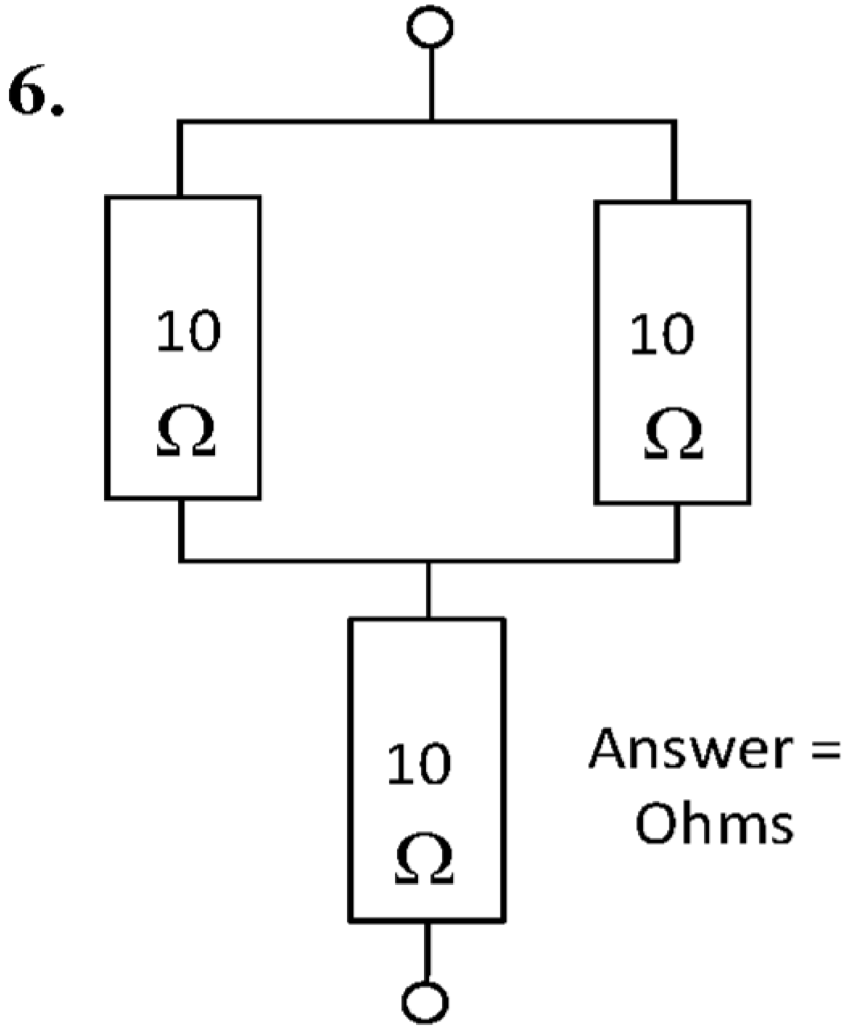
SERIES-PARALLEL CIRCUITS

The resistance is calculated as follows:

$R_T = 10 \| 10 + 10$

$R_T = 5 + 10$

$R_T = 15\ \Omega$



SERIES-PARALLEL CIRCUITS

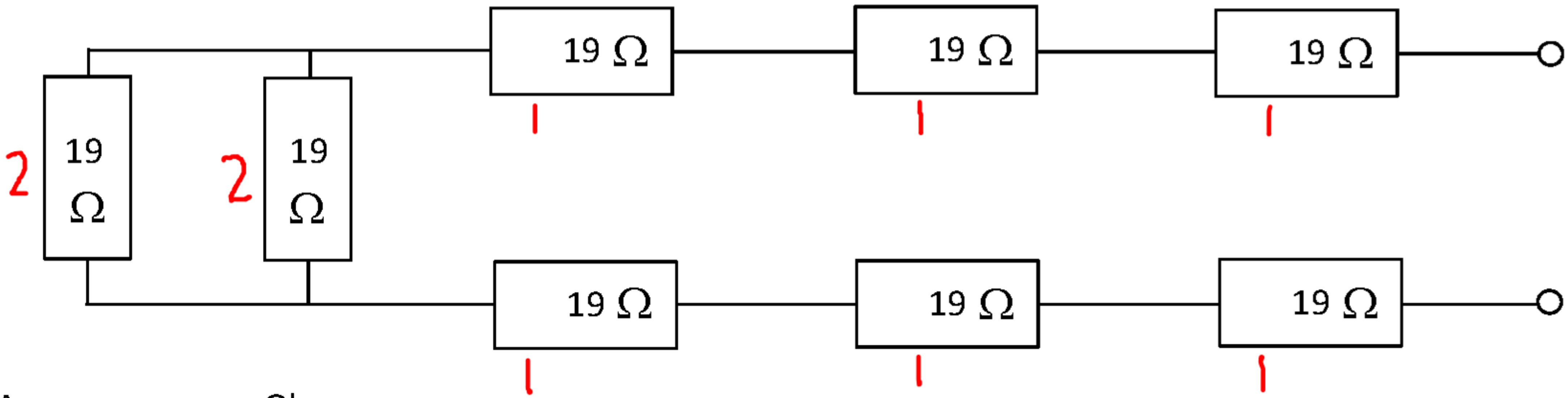
General Rules:

- *All resistors with the same current flowing through them may be put in series.*
- *All resistors with the same voltage across them may be put in parallel.*
- *A series network may consist of single resistors, and "subnetworks". The single resistors outside the subnetworks may be added up as for a simple series circuit.*
- *A parallel network may consist of single resistors, and "subnetworks". The single resistors outside the subnetworks may be added up as for a simple parallel circuit.*
- ***Do not mix series and parallel circuits in calculations! Your working must still obey Ohm's Law, and Kirchhoff's Laws!***

SERIES-PARALLEL CIRCUITS

Worksheet 10C, Q5

5.

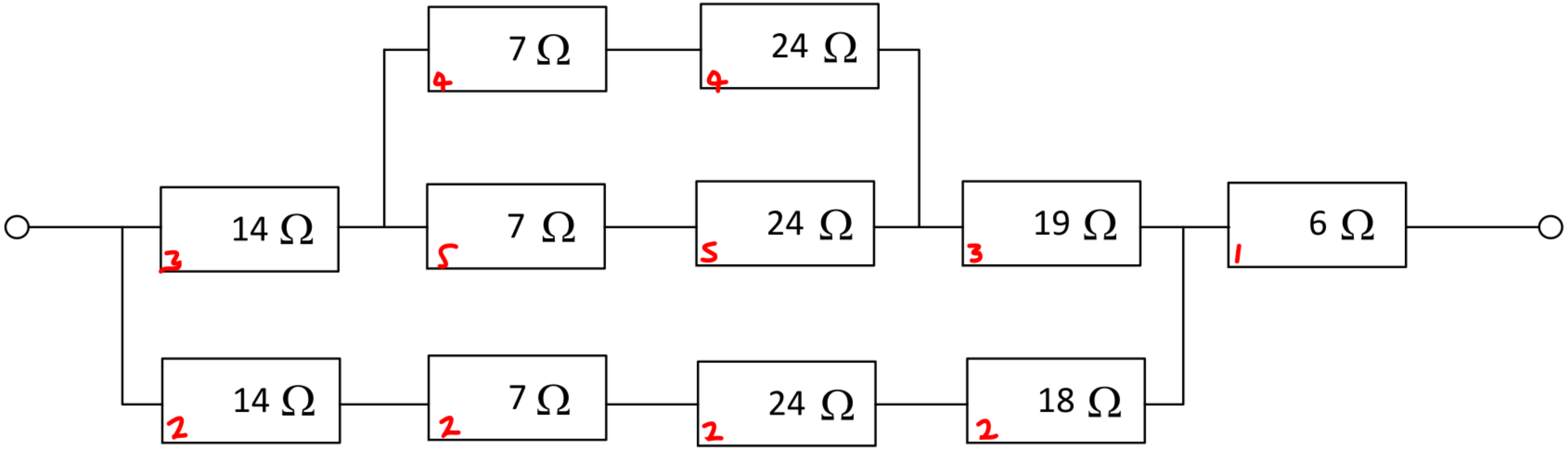


Answer = Ohms

SERIES-PARALLEL CIRCUITS

Worksheet 10D, Q4

4.



Answer = Ohms

THE END